УДК 372.851
DOI: 10.17277/voprosy.2016.03.pp.104-111

FORMATION OF COMPETENCE OF MATHEMATICAL MODELING IN THE SYSTEM "SCHOOL – HIGHER EDUCATIONAL INSTITUTION"

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Keywords: classification of models; components of competence of mathematical modeling; conception of development of mathematical education.

Abstract: The concept of competence of mathematical modeling is adapted to the process-system "school – higher educational institution". The main structural components of competence are distinguished. The classification of the models which are used in the process of learning in secondary school and characteristics of formation of competence levels are offered.

Introduction

The present system of national mathematical education is characterized by the change from knowledge-oriented educational paradigm to competence ones. Mathematics is a powerful means for the formation of subject and metasubject competencies. According to the Conception of development of mathematical education in the Russian Federation (further – the Conception, see [1].) "Studying and teaching of mathematics, on the one hand, provides readiness of learners for mathematics application in other areas, on the other hand, has a backbone function, influencing significantly intellectual readiness of school and university students for training, and understanding the content of other subjects". Among the main problems of development of mathematical

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education in the Russian Federation it is noted the need of forming applied abilities of school students using a mathematical approach to reasoning, basing, argumentation, planning, spatial constructions and numerical assessments. Therefore, the formation of the competence of mathematical modeling meets the goals set the Conception.

In the given work:

1) the components of competence of mathematical modeling are specified and agreed with the problems of the Conception;

2) the classification of models, which are used in the process of learning in secondary school, is offered and the conception of "internal mathematical modeling" is formulated;

3) the levels of learning of competence are offered.

The competence of mathematical modeling is relatively recently introduced in teaching science (see e.g. [2]); with regard to training process in secondary school, it can be defined *as ability to update and apply mathematical knowledge and skills in constructing, analyzing and interpreting mathematical models in the process of solving problems* both educational and practical. We will consider this competence in the context of its formation in the system "school – higher educational institution" as a substantive educational competence (see [3]).

We distinguish the following components of this competence.

1. *Motivational-value attitude to mathematical knowledge and the ability to build the appropriate models in the process of training and practice.* The motif is born with understanding of the universality of mathematical language, the need for the formalization of the laws of physics, chemistry, biology, economics. Studying the data and other disciplines, a learner comes to the conclusion that mathematical methods serve as a "tool" of research in various areas of activity, development, analysis and making decisions, that's why the development of mathematical disciplines has to become a conscious purpose and shall be included in the personal semantic context of its activity.

2. *Outlook and its continuously expanding* is a necessary component of the competence of mathematical modeling. It is not just a question of mastering the content of academic disciplines, but also of constant growth of the cultural level of a learner. Interest in politics, economics, history, literature, art is inevitably accompanied by an analysis of the phenomena and processes, comparative characteristics, logical reasoning etc. Appointed forms of mental activity in their turn contribute to the development of abilities to allocate the main and to reject the secondary, briefly and clearly express their ideas, assign tasks, receive and articulate conclusions, and these skills successfully "integrate" into processes of mathematical modeling.

3. *Knowledge and skills* both in the field of mathematics and in the "initial" subjects are the most essential components of this competence. First of all, it is a question of the ability to update the mathematical knowledge in relation to the alignment model in a particular situation. The use of mathematical modeling suggests:

– possession of problem-solving algorithm (question of the problem – analysis of problem situation and finding relevant information – nomination of hypotheses – building and implementation of the solution algorithm – receiving the response and the wording of the relevant conclusions);

– possession of methods to "mathematization" of objects and processes (formalization of the problem, the use of an adequate mathematical apparatus, the interpretation of solution);

– ability to build a system of arguments and justifications;

– communication skills (use in speech and writing mathematical language subjects, charting, charts, diagrams and others);

– ability to apply modern information technologies.

4. *Experience of activity* in the field of modeling assists the transfer of mathematical knowledge and skills to unknown situations, including those that arise in practice.

5. Finally, *reflection* as self-esteem activities in the field of mathematical modeling is an essential component of appropriate competence and contributes to the development of the learner's qualities as self-control, responsibility, rationality, independence.

Classification of models. In the work [4] the following three classes of models are offered using in the process of solving problems of interdisciplinary and practice-oriented trend.

1. Models of formal-logical type: it takes place here the formalization of reasoning propositional by means of propositional algebra and logic of predicates.

2. Analytical models: here the processes of functioning of real objects or systems are written as explicit functional dependencies. Moreover we distinguish:

– models-transformations, models-equations (algebraic, transcendental, differential, integral) and models-inequalities;

– models-approximations (interpolation/ extrapolation problems, numerical integration, numerical methods for solving simple differential-equations);

– models-optimizations (for example, a linear programming problem).

3. Geometric models which using planar and spatial geometric objects.

4. Probabilistic and statistical models (probabilistic characteristics of random events, analysis of statistical data and their statistical processing).

5. Mixed type models. For example, the situations which are simulated in the form of solid geometry problems, and can be solved by vector-coordinate method, they use, in fact, both analytical and geometric apparatus; problems of finding the probability of random events involve the use of logical operations (where a common basis is a Boolean algebra), numerical characteristics of random variables involve the use of analytical apparatus.

"Intrasubject" modeling. By the conception of A. A. Lyapunov [5] intrasubject ("internal mathematical") modeling will be understood as *mediated theoretical study of mathematical object (e.g. proving assertion, solving problem etc.) using a model of one of the above types which can replace it in certain relationships and giving in its study ultimately information about the modeled object*. The process of internal mathematical modeling contributes to the formation of ideas about the system of mathematical methods and techniques, expands the range of the use problem-solving tools by learners.

So the decision of geometric problems in many ways is facilitated with the involvement of analytical models (geometric problem – introduction of the unknowns – restrictions on their values-establishment connections between the known and unknown quantities – preparation of the equation or the system of equations and obtaining the compliance of relevant decisions in the framework of the analytical model – geometric interpretation). Solutions of problems of algebra or analysis in their turn are largely clarified by using models of formal-logical type (equations-investigations, equivalent to the equation as, respectively, the investigations of predicates and equivalent predicates; the system of inequalities as a conjunction, aggregate inequalities – as a disjunction of predicates and others).

Further, some algebraic raised and complex problems (e.g. problems with parameters) can be solved by means of geometric modeling. An example is the following task.

Find all values of the parameter *a* for which the system of equations

$$\begin{cases} (x+1)^2 + (y-1)^2 = 1; \\ (x-2)^2 + (y+2)^2 = a, \end{cases}$$

has a unique solution.

It's worth here to prefer the consideration geometric patterns, namely situations of internal and external tangency of two circles (the system is recorded in the form of equations) to standard methods of solution (methods of substitution or algebraic addition), then the parameter *a* is determined by applying the Pythagorean theorem.

We will use the so-called **competence passport** to clarify the results of development of mathematical modeling in terms of "to know / to be able / to master". It comprises:

– the place and the importance of competence in accordance with the requirements of the Federal State Standard and also states of the Conception for formation of the level of competence at the end of the development of the General Educational Programmes (**GEP**);

– clarify of components of the competence content;

– structuring of competence into levels, indicators and descriptors.

1. *Place and significance of the competence of mathematical modeling in the overall expected result of the formation of the graduate of school.*

Formation of this competence is an important factor in preparing graduates to perform the following types of training and practice:

– analysis of concepts, facts and situations from a variety of disciplinary areas using logical conclusions, mathematical language and methods of mathematics and obtaining, therefore, necessary information within the relevant disciplinary area;

– interpolation and extrapolation of the results;

– prognosis of behavior of processes by means of the probabilistic and statistical theory;

– receipt, ultimately, practical advice in solving of applied problems.

2. *Specified components of competence content:*

– subject – the theoretical basis of the competence of mathematical modeling, comprising mathematical knowledge and skills as well as appropriate modes of action: the use of methods of mathematical logic, geometry, algebra, calculus and stochastics;

– the actual model, providing translation conditions of the problem into mathematical language; in particular, the use of mathematical language to write the simplest laws and facts from the field of natural sciences, economic and other disciplines;

– computer – problem solving in general and with specific numerical values of quantities, implying knowledge of the rules and laws of computing and transforms the model;

– predictive – it aims at clarifying the development trends with the states of researched phenomenon or object.

3. *Results of learning, revealing the structure of competence and its planned levels of formation.*

We distinguish the following three main levels.

1. The threshold level is the *minimum required* for all graduates of secondary school on completion of the GEP. At this level it is sufficient to master such a minimal system of knowledge, skills for the analysis of simple mathematical models. The work with more complex models is carried out under the guidance of a teacher.

2. The basic level allowing to solve *standart problems*, to use known algorithms, rules and methods both in solving mathematical problems itself and on the stages of mathematical modeling. It is, in fact, a question of accordance to the requirements for the results of the development of the GEP of secondary (full) general education in mathematics at the basic level.

3. The advanced level – *the highest possible expression of competence* which is important as a quality benchmark for self-improvement. We are talking about compliance to the development of the GEP to the results of secondary (full) general education in mathematics at the profile and depth levels. Here are the main (in our opinion) features of each of the renumbered levels (to know /to be able to / to master).

At the threshold level a learner needs

know:

– main formulas algebra and trigonometry;

– definitions and graphics of basic elementary functions;

– formulation of the concept facts of geometry needed to solve simple planimetric and stereometric problems;

– concepts and facts from mathematical analysis which are necessary for the study of functional connections;

– elementary provisions of the probability theory and mathematical statistic;

able to:

– carry out standard algebraic and trigonometric transformation and solve simple algebraic and transcendental equations;

– visualization of data as a geometric object;

– calculate derivatives and integrals based on tabulated formulas;

– extract information presented in tables, charts, graphs;

master:

– the methods of geometrical interpretation of the simplest tasks of determination the relative location of objects and their sizes and numerical characteristics (area, volume);

– the methods of differential-integral calculus in the simplest cases, the study models (e.g. the calculation of speed, square, variable work force etc.);

– ways of systematization of statistical data in the form of a series of distribution, polygons and histograms.

At the basic level the student must

know:

– formulas of algebra and trigonometry fully used in school mathematics, functional concepts and their graphical interpretation as well as the concept and the facts of geometry;

– the provisions of the differential-integral calculus needed to study the functions and location of geometrical and physical quantities;

– main concepts, probability charts and formulas of the theory of random events, as well as the concepts and facts related to the analysis of empirical distributions;

be able to:

– carry out the proof of the main known (from the school course) mathematical statements;

– perform the basic steps of the algorithm of mathematical modeling (formalization of the problem, study models, interpretation of results);

– use functional graphical representations for solving both mathematical and practical-oriented problems;

– use facts to describe the geometry of objects of the world;

– describe and analyze numerical data arrays using right statistical characteristics, use the understanding of probabilistic properties of the surrounding phenomena in decision-making;

master:

– skills of oral, written, instrumental calculations for the solution of practical problems;

– the symbol language of algebra, trigonometry, geometry, and its use in the formalization of the problems and related areas of practice-oriented problems, and the need for their solution techniques perform transformations, finding roots of equations etc .;

– the system of functional concepts and facts to describe and analyze the real dependencies;

– ways of presentation and analysis of statistical data; ways of studying the statistical regularities in the real world, ways of building and studying of simple probabilistic models.

At the advanced level it is assumed that the learner, in addition to knowledge which he developed at the basic level, has the primary knowledge of analytic geometry, complex analysis, the theory of polynomials, differential equations, and also knows how to combine known methods of proof in justifying the new allegations, to carry over learned techniques of problem solution on new ones, including the practical situation.

The advanced level assumes that a school student masters:

– special techniques to solve highly complex problems (e.g. problems with parameters etc.);

– vector-coordinate method (in addition to the standard geometrycal methods) analyzing the location of objects on the plane and in the space;

– advanced algorithm research of functional dependencies (including the asymptotic behavior of the functions, the nature of the convexity etc.) and special methods of integration of certain classes of functions;

– standard distributed random variables and methods of receipt of point and interval estimates of parameters of theoretical distributions.

Emergence. It seems that the process of mathematical modeling, of at each stage generates the following system effects.

1. The formulation of the problem of processes and phenomena modeling is *motivated* to find ways of formalizing application problems, the development of a mathematical language, and existing relevant mathematical facts and methods.

2. The research process of the mathematical model serves as an incentive to expand the arsenal of *means of dealing with the actual mathematical problems*.

3. On the stage of results interpretation of the model study there is *an understanding of the universality* of mathematical language and mathematical methods that are encouraged to go beyond the original problem and the spread of the simulation results to other disciplinary areas.

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Формирование компетенции в области математического моделирования в системе «школа-вуз»

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Ключевые слова: классификация моделей; компоненты компетенции математического моделирования; концепция развития математического образования.

Аннотация: Понятие компетенции математического моделирования адаптировано к процессной системе «школа-вуз». Выделены основные структурные компоненты компетенции. Предложены классификация моделей, используемых в процессе обучения в старшей школе и характеристики уровней сформированности компетенции.
