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DEVELOPMENT OF ONE-DIMENSIONAL MODEL OF ARTERIAL TREE FOR MULTISCALE MODEL OF HEMODYNAMICS FOR RESEARCH OF CEREBRAL CIRCULATION

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Abstract: The arterial tree model, including description of the upper-body arteries and detailed description of cerebral circulation, is proposed. The model can be used for determination of the hemodynamic parameters in the circle of Willis. The simulation results allows for correct definition of the boundary conditions for the multiscale hemodynamic models.

For research of the cerebral circulation it is proposed to use a multiscale mathematical model of hemodynamics, consisting of the set of mathematical models of circulation with a different level of detail [1, 2].

Mathematical model of the arterial tree is used for coupling of the model of global hemodynamics (0D model) [3] with the model of local hemodynamics of a cerebral artery. In the developed model, the arterial tree is described as set of one-dimensional arteries. The following assumptions are used: blood velocity and pressure changes only in one dimension (along the vessel); blood is modeled as incompressible Newtonian fluid; flow is laminar; vessel wall is isotropic, linear-elastic; vessel wall is incompressible; gravity is neglected. The 1D model structure, including 48 main arteries, is shown in Fig. 1. The model includes upper-body arteries and a detailed description of the cerebral circulation [4].

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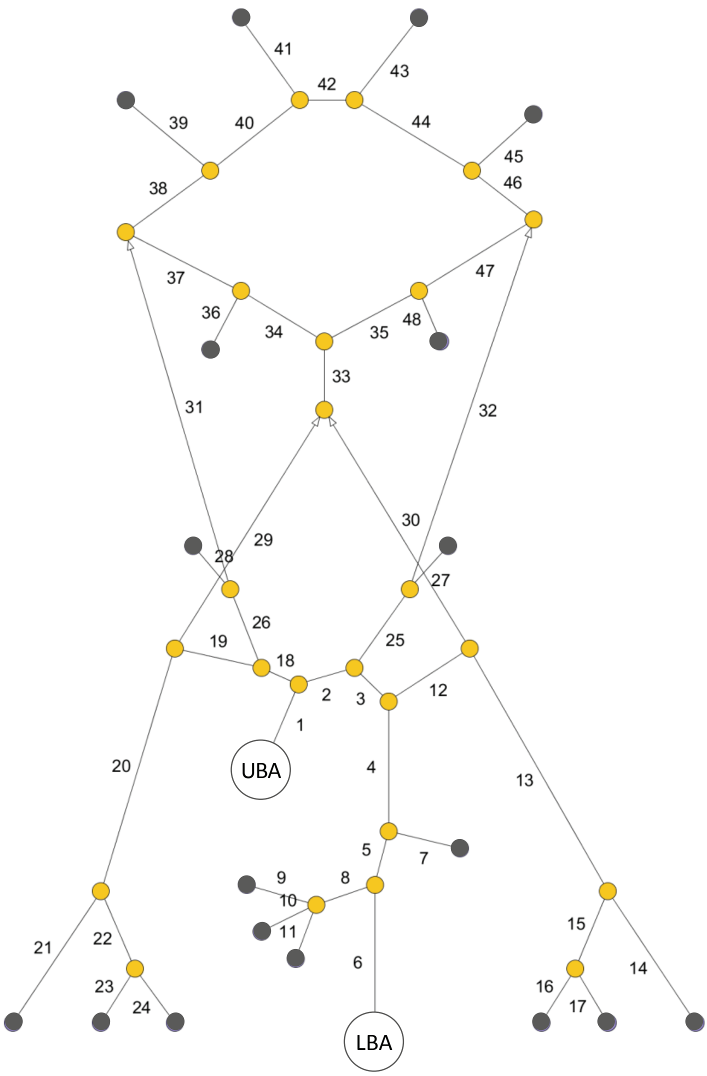

Fig. 1. The structure of the one-dimensional mathematical model of the upper-body and cerebral arteries. Red color is used to represent the nodes of the 0D model, yellow color is used for bifurcations; violet color is used for terminal elements

Sample element of the one-dimensional artery is shown in Fig. 2. In one dimensional representation *i*-th vessel with the length L_i is divided in n_i elementary segments with the length dx

$$n_i = \frac{L_i}{dx}, \qquad i \in [1, N], \tag{1}$$

where n_i is the number of the elementary segments in *i*-th vessel; L_i is the length of the *i*-th vessel; dx is the length of the elementary segment of the vessel; N is the total number of arteries in the arterial tree model.

The elementary segment is characterized by the elementary blood volume dV and the pressure dP inside it. The link between *i*-th and *i*-1-th elementary segments is characterized by volumetric blood flow $q_{i-1,\,i}$.

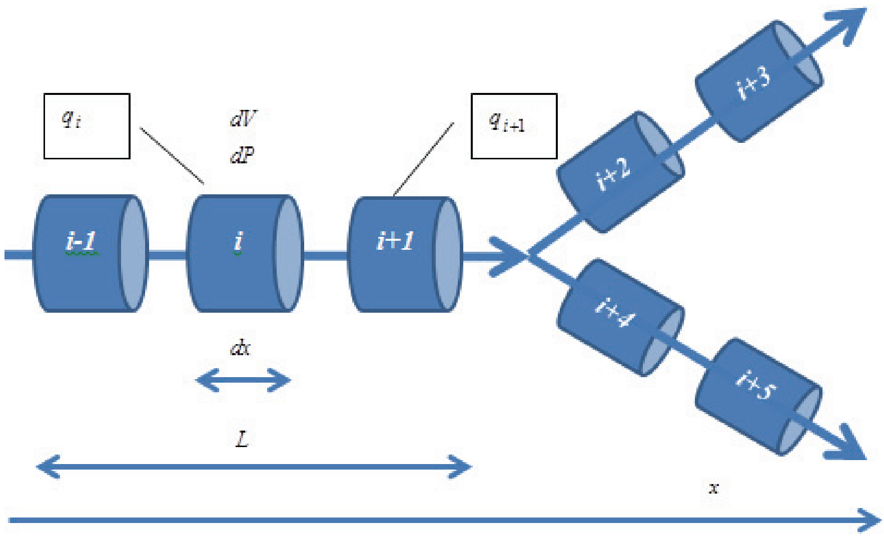

Fig. 2. A schematic representation of the vessel and bifurcation

For the elementary volume dV_i, it can be written

$$\frac{dV_i}{dt} = \sum_{\text{in}} q_{\text{in}\,i} - \sum_{\text{out}} q_{\text{out}\,i}, \tag{2}$$

where $\sum_{\text{in}} q_{\text{in}\,i}$ is the sum of the volumetric blood flows going in to the *i*-th elementary segment; $\sum_{\text{out}} q_{\text{out}\,i}$ is the sum of the volumetric blood flows going out the *i*-th elementary segment.

For the pressure dP_i in the elementary segment, it can be assumed that the linear dependency between elementary volume dV_i and pressure dP_i

$$dP_i = e_i (dV_i - dU_i), \tag{3}$$

where dU_i is the unstrained volume of the *i*-th elementary segment; e_i is the wall elasticity of the *i*-th elementary segment.

For the volumetric blood flow q_i, the Poiseuille law can be used:

$$q_i = \frac{\pi d_i^{\,4}}{128\eta L_i} \Delta P_i, \tag{4}$$

where η is thedynamic viscosity; d_i is thediameter of the *i*-th elementary segment.

For the cross-section area, the following assumption is used:

$$dS_i = \pi r_i^2 = \frac{dV_i - dU_i}{dx}, \tag{5}$$

where dS_i is the area of the *i*-th segment cross-section; r_i is the radius of the cross-section.

For the elementary segment, the following equations can be used:

$$\begin{cases} \dfrac{dV_i}{dt} = q_{i-1} - q_i; \\ q_i = \dfrac{P_i - P_{i+1}}{R_i}; \\ P_i = C_i(V_i - U_i). \end{cases} \quad (6)$$

After some operations, the final equation for the elementary segment is as follows:

$$\frac{dVi}{dt} = \left[\frac{C_{i-1}}{R_{i-1}}V_{i-1} + \frac{\left(-R_i - R_{i-1}\right)C_i}{R_{i-1}R_i}V_i + \frac{C_{i+1}}{R_i}V_{i+1}\right] + \\ + \left[-\frac{C_{i-1}U_{i-1}}{R_{i-1}} + \frac{\left(R_i + R_{i-1}\right)C_iU_i}{R_{i-1}R_i} - \frac{C_{i+1}U_{i+1}}{R_i}\right] \quad (7)$$

and in the matrix form:

$$\begin{aligned} &\frac{dV_i}{dt} = \mathbf{A}_i\mathbf{V}_i + \mathbf{B}_i; \\ &\mathbf{A}_i = \begin{bmatrix} \dfrac{C_{i-1}}{R_{i-1}} & \dfrac{\left(-R_i - R_{i-1}\right)C_i}{R_{i-1}R_i} & \dfrac{C_{i+1}}{R_i} \end{bmatrix}; \\ &\mathbf{V}_i = \begin{bmatrix} V_{i-1} \\ V_i \\ V_{i+1} \end{bmatrix}; \\ &\mathbf{B}_i = \begin{bmatrix} \dfrac{-C_{i-1}U_{i-1}}{R_{i-1}} + \dfrac{\left(R_i + R_{i-1}\right)C_iU_i}{R_{i-1}R_i} - \dfrac{C_{i+1}U_{i+1}}{R_i} \end{bmatrix}. \end{aligned} \quad (8)$$

Spreading the presented method for the set of elementary segments, the following system can be obtained:

$$\begin{aligned} &\frac{d\mathbf{V}}{dt} = \mathbf{AV} + \mathbf{B}; \\ &\mathbf{A} = \begin{bmatrix} \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dfrac{C_{i-1}}{R_{i-1}} & \dfrac{\left(-R_i - R_{i-1}\right)C_i}{R_{i-1}R_i} & \dfrac{C_{i+1}}{R_i} & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \dots \\ V_n \end{bmatrix}; \\ &\mathbf{B} = \begin{bmatrix} \dots \\ \dots \\ \dfrac{-C_{i-1}U_{i-1}}{R_{i-1}} + \dfrac{\left(R_i + R_{i-1}\right)C_iU_i}{R_{i-1}R_i} - \dfrac{C_{i+1}U_{i+1}}{R_i} \\ \dots \\ \dots \end{bmatrix}. \end{aligned} \quad (9)$$

In the developed model the unknowns are q_{i-1} and P_{i+1}. These values can be obtained from the 0D model [5]. Thus for the inlet segment where q_0 is known, the following equation can be used:

$$\frac{dV_1}{dt}=\left[-\frac{C_1}{R_1}V_1+\frac{C_2}{R_1}V_2\right]+\left[\frac{C_1U_1-C_2U_2}{R_1}+q_0\right]. \tag{10}$$

For the outlet segment of the arterial tree where P_{n+1} is known, the equation will be:

$$\frac{dV_n}{dt}=\left[\frac{C_{n-1}}{R_{n-1}}V_{n-1}+\frac{(-R_n-R_{n-1})C_n}{R_{n-1}R_n}V_n\right]+$$
$$+\left[-\frac{C_{n-1}U_{n-1}}{R_{n-1}}+\frac{(R_n+R_{n-1})C_nU_n}{R_{n-1}R_n}+\frac{P_{n+1}}{R_n}\right]. \tag{11}$$

For bifurcations it can be written:

$$\frac{dV_{i+1}}{dt}=\left[\frac{C_i}{R_i}V_i-\frac{C_{i+1}}{R_i}V_{i+1}+\frac{C_{i+2}}{R_{i+1}}V_{i+2}-\frac{C_{i+4}}{R_{i+1}}V_{i+4}\right]+$$
$$+\left[-\frac{C_iU_i}{R_i}+\frac{C_{i+1}U_{i+1}}{R_i}-\frac{C_{i+2}U_{i+2}}{R_{i+1}}+\frac{C_{i+4}U_{i+4}}{R_{i+1}}\right]. \tag{12}$$

Thus, the blood flow in the one-dimensional model can be modeled using equations (1) – (12).

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Разработка одномерной модели артериального русла с учетом ее использования в многомасштабной модели гемодинамики для исследования церебрального кровообращения

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Ключевые слова: гемодинамика; математическая модель; мозговое кровообращение; сердечно-сосудистая система.

Аннотация: Предложена модель артериального русла, включающая описание артерий верхней части тела и детальное описание церебрального кровообращения. Данная модель позволяет рассчитать гемодинамические параметры в области круга Виллиса, что может быть использовано для определения граничных условий в многомасштабных моделях гемодинамики.
