УДК 612.13.001.575

MATHEMATICAL MODELING OF BLOOD FLOW IN BASILAR ARTERY BIFURCATION REGION*

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Key words and phrases: aneurysm; basilar artery; cardiovascular system; hemodynamics; mathematical model.

Abstract: Basilar artery aneurysm occurrence is associated with hemodynamics instability. For prediction of aneurysm various experimental and numerical methods are developed. However, the greatest interests are mathematical methods for computing the hemodynamic parameters at the bifurcation of the basilar artery. In this paper, we propose a partial derivatives mathematical model of hemodynamic to compute the three-dimensional blood flow in the basilar artery. The model is possible to compute the basic hemodynamic parameters of blood flow and find the value of wall shear stress, which has a significant influence on the formation and development of aneurysms of the basilar artery. The developed model can also be used in multiscale models of hemodynamics to combine the hemodynamic models of different levels of detail.

Introduction. Basilar artery, located in the pons, formed by the junction of the vertebral arteries. The main disease of basilar artery is aneurysm which is formed at the bifurcation region of basilar artery (Fig. 1). Currently, the causes of aneurysm development are not fully understood. It is assumed that the main factors contributing to the formation of aneurysms are hemodynamic instability and geometrical parameters of the basilar artery [1]. As a result of hemodynamic instability in basilar artery there is a region with a low wall shear stress, which leads to the destruction of the inner layer of the vessel wall and the loss of elasticity [2]. As a result, in the vessel with a weakened wall aneurysm is formed. Especially dangerous is a rupture of the aneurysm, which leads to subarachnoid hemorrhage.

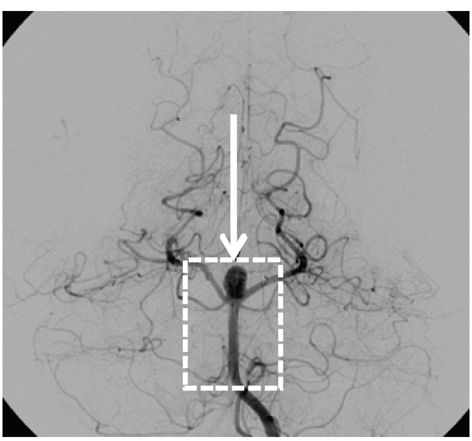


Fig. 1. Angiogram presents the basilar artery aneurysm [3]

One of the topical tasks today is to develop methods for predicting the emergence and development of aneurysms of the basilar artery. The most

* The reported study was supported by the Supercomputing Center of Lomonosov Moscow State University [14].

promising method for such problems is the use of mathematical modeling of blood flow in the basilar artery.

Methods. Strictly speaking, the blood is a suspension, however, given that the diameter of the basilar artery is relatively large (> 1 mm), it can be accepted that blood is a Newtonian fluid. Vessel diameter size of 1 mm. is critical, because for vessels with diameters smaller than 1 mm observed Fahraeus–Lindqvist effect of or sigma effect [4, 5].

To describe the three-dimensional blood flow as incompressible Newtonian fluid we use the conservation momentum law [6]:

$$\frac{\partial u_x}{\partial t}+u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}+u_z\frac{\partial u_x}{\partial z}-\nu\left(\frac{\partial^2 u_x}{\partial x^2}+\frac{\partial^2 u_x}{\partial y^2}+\frac{\partial^2 u_x}{\partial z^2}\right)+\frac{1}{\rho}\frac{\partial P}{\partial x}=f_x;$$

$$\frac{\partial u_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}+u_z\frac{\partial u_y}{\partial z}-\nu\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}+\frac{\partial^2 u_y}{\partial z^2}\right)+\frac{1}{\rho}\frac{\partial P}{\partial y}=f_y;$$

$$\frac{\partial u_z}{\partial t}+u_x\frac{\partial u_z}{\partial x}+u_y\frac{\partial u_z}{\partial y}+u_z\frac{\partial u_z}{\partial z}-\nu\left(\frac{\partial^2 u_z}{\partial x^2}+\frac{\partial^2 u_z}{\partial y^2}+\frac{\partial^2 u_z}{\partial z^2}\right)+\frac{1}{\rho}\frac{\partial P}{\partial z}=f_z$$

or in vector form:

$$\frac{\partial u}{\partial t}+u\nabla u-\nu\Delta u+\frac{1}{\rho}\nabla P=f,\qquad \nu=\frac{\mu}{\rho},\tag{1}$$

where u – blood velocity, m/s; ν – kinematic blood viscosity, m^2/s; μ – dynamic blood viscosity, Pa/s; ρ – blood density, kg/m^3; P – blood pressure, Pa; f – external forces; t – time, s.

Under external forces f in equation (1) is usually understood gravity. This value in the simulation of hemodynamics, tend to neglect ($f = 0$). Blood kinematic viscosity consider constant $\nu = \text{const}$. Thus, in equation (1) there are two unknowns: blood velocity u and pressure P. Consequently, it is needed to add one more equation. This equation is continuity equation

$$\frac{\partial\rho}{\partial t}+\operatorname{div}\rho u=0.\tag{2}$$

Since blood is modeled as incompressible Newtonian fluid, then $\rho = \text{const}$. Therefore, equation (2) can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$$

or in the vector form

$$\operatorname{div} u=0.\tag{3}$$

Thus, equations (1), (3) formed a closed equation system, describing blood flow through basilar artery in 3D (3D hemodynamics model).

Consider the computational domain D (Fig. 2). Computational domain consists of basilar artery and two vessels in which basilar artery bifurcates. According to experimental data, length l_{BA} of basilar artery vary from 24.8 to 38.5 mm. Diameter d_{BA} of basilar artery vary from 2.7 to 4.28 mm. In bifurcation region basilar artery bifurcates on left and right posterior cerebral arteris. Bifurcation angle α vary from 30° to 180°. Length of left/right posterior cerebral artery (l_{left}/l_{right}) is $(6.9\pm3.1)/(6.8\pm2.7)$ mm respectively. Diameter of left/right posterior cerebral artery (d_{left}/d_{right}) is $(2.2\pm0.6)/(2.1\pm0.7)$ mm respectively [7].

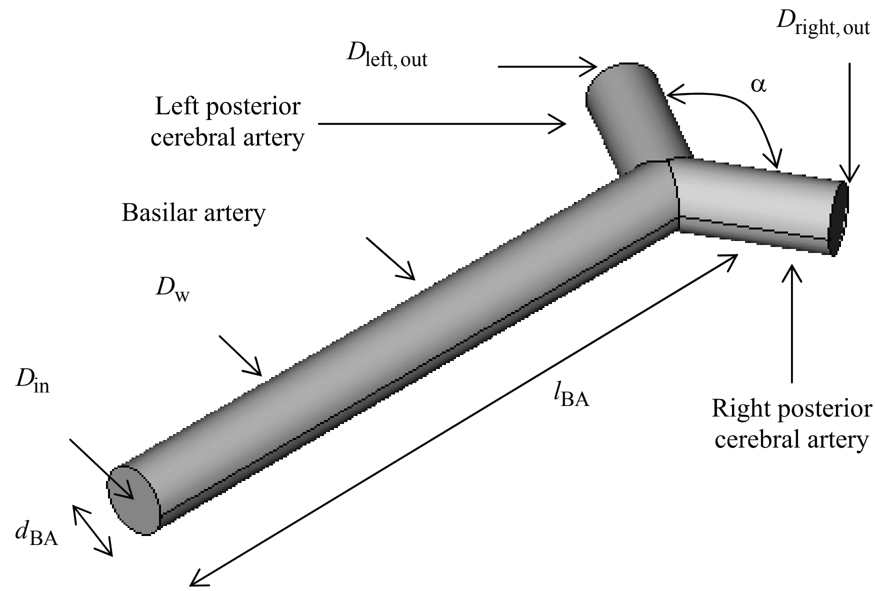


Fig. 2. Geometrical 3D model of basilar artery

Basilar artery walls are elastic and can be stretched under the influence of moving blood on them. The increment in the radius of the vessel reaches approximately 15 % of the original [4]. However, for the basilar artery radius increment is negligible, therefore, the elasticity of the basilar artery can be neglected and considered them rigid.

For solving system of equations (1) – (3) it is necessary to apply proper initial and boundary conditions.

For the vessel wall D_{w} it is no-slip condition

$$u\big|_{D_{\text{w}}} = 0. \tag{4}$$

For inlet boundary D_{in}, and also for outlet boundaries $D_{\text{left, out}}$, $D_{\text{right, out}}$ boundary conditions may be applied by researcher. However most promising method to apply boundary conditions is to obtain them from multiscale hemodynamics models [4, 8]. In this case simple hemodynamics models [9, 10] are used, for example global hemodynamics model of arterial tree model. Such models are less computationally costly, however only lumped parameters of blood flow can be found. As a rule, to get the boundary conditions for 3D model it is used results of artery tree model (1D model), converted according to special coupling algorithm (Fig. 3). Thus:

$$u\left(x_{\text{in}}, y, z, t\right)\Big|_{D_{\text{in}}} = A\left(u_{1D}(x_{\text{in}}, t)\right);$$

$$u\left(x_{\text{left, out}}, y, z, t\right)\Big|_{D_{\text{left, out}}} = A\left(u_{1D}(x_{\text{left, out}}, t)\right);$$

$$u\left(x_{\text{right, out}}, y, z, t\right)\Big|_{D_{\text{right, out}}} = A\left(u_{1D}(x_{\text{right, out}}, t)\right),$$

where x_{in}, $x_{\text{left, out}}$, $x_{\text{right, out}}$ – coordinates along the vessel, where 3D and 1D models coupling; u_{1D} – value of blood velocity, computed by arterial tree model; A – coupling algorithm.

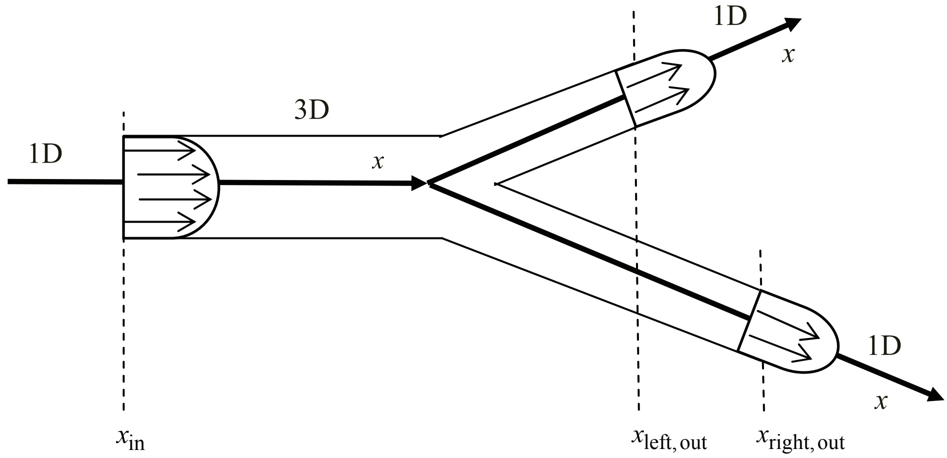


Fig. 3. Scheme of coupling of 3D basilar artery model and 1D arterial tree model

To apply proper initial conditions $u_0 = u(x, y, z, t)$ и $P_0 = P(x, y, z, t)$ when $t = 0$ it is needed to solve Stokes problem

$$\begin{cases} -\nu\,\Delta u_0 + \nabla P_0 = f; \\ \operatorname{div} u_0 = 0. \end{cases} \tag{5}$$

In system (5) also as in the system (1), (3) influence of external forces is neglected ($f = 0$). The boundary conditions are the same as for system (1) – (3).

Results. To conduct numerical experiments computational domain D was constructed (Fig. 2). Parameters of computational domain are: $l_{BA} = 30$ mm; $d_{BA} = 3$ mm; $\alpha = 120°$; $l_{left} = 7$ mm; $l_{right} = 7$ mm; $d_{left} = 2$ mm; $d_{right} = 2$ mm. Blood parameters are: $\nu = 3.3\cdot10^{-6}$ m^2/s; $\mu = 0.003$ Pa/s; $\rho = 1050$ kg/m^3.

For solving the mathematical model it was used Finite Element Method (**FEM**). Computational domain was divided on 18984 finite elements (tetrahedrons). In basilar artery Reynolds number Re vary from 200 to 600 [11], hence, it can be concluded that there is a laminar flow in basilar artery. For modeling of blood flow through basilar artery on inlet boundary D_{in} velocity profile was applied according to Poiseuille's law [12] (Dirichlet condition):

$$u(x_{in}, y, z, t)\big|_{D_{in}} = u_{max}(t)\left(1 - \frac{r(y,z)^2}{\left(d_{BA}/2\right)^2}\right);$$
$$u_{max}(t) = 2u_{mn}(t); \tag{6}$$
$$r(y,z) = (y - y_0)^2 + (z - z_0)^2,$$

where y_0, z_0 – coordinate of center of D_{in}; u_{mn} – mean blood velocity; u_{max} – maximum blood velocity.

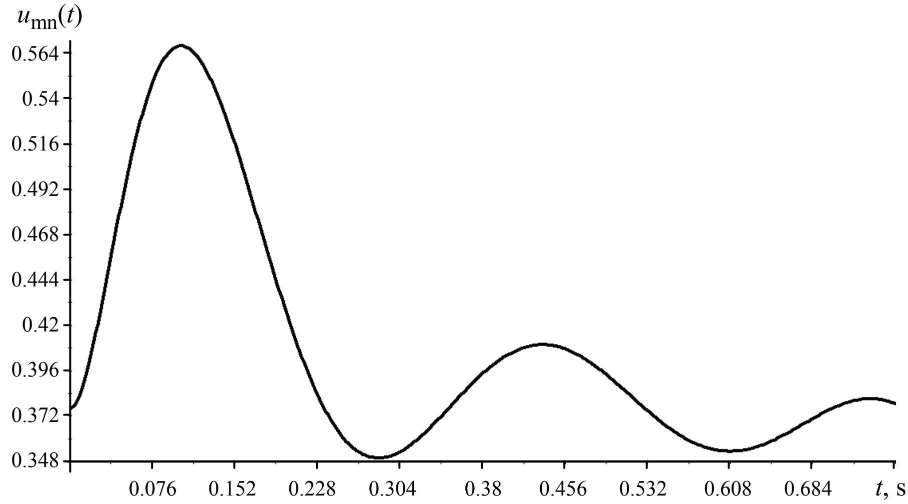


Fig. 4. Mean inlet blood flow velocity in basilar artery

According to [13] mean blood velocity in basilar artery is 0.397 m/s. For modeling it was used the following function of mean (Fig. 4).

For outlet boundaries $D_{\text{left, out}}$, $D_{\text{right, out}}$ was applied free flowing conditions (Neumann condition):

$$\begin{aligned} \left.\frac{\partial u}{\partial n}\right|_{D_{\text{left, out}}} &= 0; \\ \left.\frac{\partial u}{\partial n}\right|_{D_{\text{right, out}}} &= 0, \end{aligned} \tag{7}$$

where n – external normal to $D_{\text{left, out}}$, $D_{\text{right, out}}$.

For vessel wall was applied the no-slip condition (4).

On C++ programming language was developed software for computing system of equations of mathematical model of blood flow through basilar artery (1), (3) – (7) with usage of MPI technology.

Solving equations (1), (3) is high computationally costly. Hence, for solving the Lomonosov supercomputer of MSU was used [14]. As of January 2014 Lomonosov supercomputer by performance takes:

– 1 place in Top50 list (supercomputers list of CIS) [15];

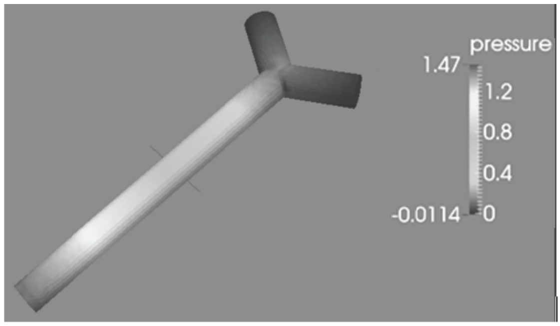
– 37 place in Top500 list (world supercomputers list) [16];

– 43 place in Graph500 (world supercomputers list, oriented on solving DIC (Data Intensive Computing) problems) [17].

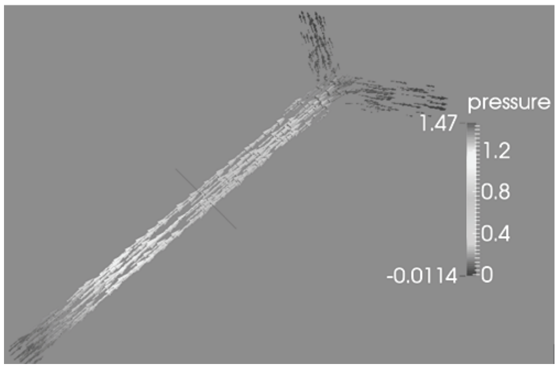
Supercomputer consists of more than 5000 computational nodes with peak performance of 1.7 Pflops. Main processor for computational nodes is Intel Xeon 5570 Nehalem [18]. CPU frequency is 2.96 ГГц, number of cores – 4, number of threads – 8. Computational nodes are joined by QDR InfiniBand bus.

Software runs with following parameters: $N = 256$, $n = 2048$, where N – numbers of computational nodes; n – number of MPI-processes.

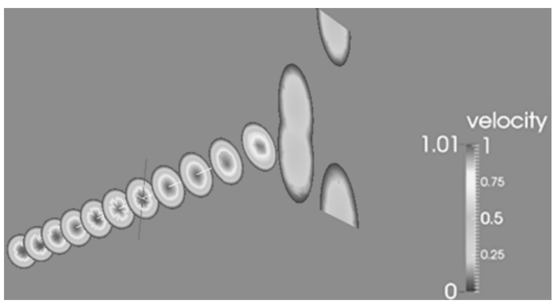
Time period of $t \in [0;1]$ s was modeled with time discretization step $dt = 0.0001$ s. Computation results show on Fig. 5.



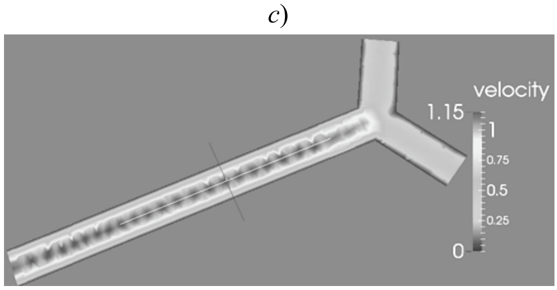
a)



b)



c)



d)

Fig. 5. Basilar artery bifurcation region:
a – pressure distribution at $t = 0$; *b* – velocity field at $t = 0$; *c* – basilar artery perpendicular slices at $t = 0.133$; *d* – basilar artery longitudinal slices at $t = 0.133$

Discussion. Thus, mathematical model of blood flow through basilar artery was developed. Using this model hemodynamics parameters and wall shear stress can be computed. This information can be used for forecasting of basilar artery aneurysm emergence and development.

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Математическое моделирование движения крови в области бифуркации базилярной артерии

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Ключевые слова и фразы: аневризма; базилярная артерия; гемодинамика; математическая модель; сердечно-сосудистая система.

Аннотация: Возникновение аневризмы базилярной артерии связано с нарушением гемодинамики. Для прогнозирования возникновения аневризмы применяются различные методы натурных и численных экспериментов. Однако наибольший интерес представляют собой математические методы расчета гемодинамических параметров в области бифуркации базилярной артерии. В работе предложена модель гемодинамики в частных производных для расчета трехмерного течения крови по базилярной артерии. С помощью модели возможен расчет основных гемодинамических параметров течения крови и вычисление пристеночного напряжения сдвига, которое оказывает существенное влияние на образование и развитие аневризмы базилярной артерии. Также возможно использование разработанной модели в многомасштабных моделях гемодинамики, позволяющих объединить модели гемодинамики разного уровня детализации.

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Mathematische Modellierung der Bewegung des Blutes auf dem Gebiet der Bifuraktion der Basilararterie

Zusammenfassung: Das Entstehen die Aneurysmen der Basilararterie ist mit der Verletzung der Hämodynamik verbunden. Für die Prognostizierung des Entstehens die Aneurysmen werden verschiedene Methoden der Außen- und numerischen Experimente verwandt. Jedoch stellen das meiste Interesse die mathematischen Methoden der Berechnung der hemodynamischen Parameter auf dem Gebiet der Bifuraktion der Basilatarterie dar. In der Arbeit ist das Modell der Hämodynamik in den privaten Ableitungen für die Berechnung der dreimeβigen Strömungen des Blutes nach der Basilararterie angeboten. Mit Hilfe des Modells ist die Berechnung der hämodynamischen Hauptparameter der Strömung des Blutes und die Berechnung der Wandanstrengungen der Verschiebung möglich, die den wesentlichen Einfluss auf die Bildung und die Entwicklung die Aneurysmen der Basilararterie leistet. Auch ist die Nutzung des entwickelten Modells in den mehrgroßzügigen Modellen der Hämodynamik, zulassend möglich, die Modelle der Hämodynamik verschiedenen Niveaus der Detaillierung zu vereinigen.

Modélage mathématique du courant de sang dans le domaine de la bifurcation de l'artère basilaire

Résumé: L'occurence de l'anévrysme de l'artère basilaire est liée à l'instabilité de l'hémodynamique. Pour la prévision de l'occurence de l'anévrysme sont employées de différentes méthodes des expériments naturels et numériques. Le plus grand intérêt présentent les méthodes mathématiques du calcul des paramètres hémodynamiques dans le domaine de la bifurcation de l'artère basilaire. Est proposé le modèle de l'hémodynamique dans les dérivées particulières pour le calcul du courant de sang par une artère basilaire. A l'aide de ce modèle est possible de calculer des essentiels paramètres hémodynamiques du courant de sang ainsi que la tension du décalage qui influence sur la formation et le développement de l'anévrysme de l'artère basilaire. Il est possible d'utiliser le modèle élaboré dans les modèles de grande échelle de l'hémodynamique permettant d'unifier les modèles de l'hémodynamique de différents niveaux des détails.

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