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TO VIBRORHEOLOGICAL EFFECTS FORECASTING DURING RAPID GRAVITY FLOW OF PARTICULATE SOLIDS

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Abstract: An experimental and analytical method is developed to forecast the profiles of velocity and the fraction of void volume in rapid gravity flows of particulate solids on a vibrated rough chute. This method assumes the analogy between particulate medium under conditions of relatively big component of fluctuation velocity of particles and the dense gas. As the first stage, the method was approved on a vibrated motionless bed of nonelastic cohesionless particles.

Symbols

A – vibration amplitude, m;	α – chute inclination angle, deg;
$b = (\pi/(6(1-\varepsilon)))^{0,33}$ – geometrical parameter;	α_0 – the angle of repose of the material, deg;
d – diameter of particles, m;	β – angle between the vibration direction and normal to the chute surface, deg;
$\frac{du}{dy}$ – shear rate, s^{-1};	ε – fraction of void volume, $m^3 \cdot m^{-3}$;
g – gravity acceleration, $m \cdot s^{-2}$;	$\bar{\varepsilon}$ – bed dilatation, $m^3 \cdot m^{-3}$;
h – bed depth, m;	ε_0 – fraction of the void volume of dense packed particles, $m^3 \cdot m^{-3}$;
k – restitution coefficient at collisions;	λ – experimental coefficient of the tangent loss at particle collision;
$p(y)$ – analogue of hydrostatic pressure, $N \cdot m^{-2}$;	μ – friction coefficient;
s – mean distance between particles, m;	ρ – particle density, $kg \cdot m^{-3}$;
T – vibration period, s;	χ – coefficient of the granular medium state equation;
t – time, s;	ω – vibration frequency, s^{-1}.
$u(y)$ – flow velocity, $m \cdot s^{-1}$;	
v' – fluctuation component of the particle velocity, $m \cdot s^{-1}$;	
y, x_1 – cartesian coordinates;	

It's well known that vibration is the most effective means to control the non-uniform particle behavior in particulate medium flows [1, 2]. Evidently, in order to forecast this behavior it is necessary to estimate vibrorheological effects i.e. to find the answer to the question how structural and kinematical flow characteristics change by the vibration action.

However, many theoretical and experimental studies have been carried out to reveal the features of particle interactions in horizontal vibrated beds [3 – 6] in spite of very big practical interest in vibrated shear flows.

In the present paper the mentioned vibrorheological effects are analyzed and an experimental and analytical method is proposed to determine the profiles of velocity and the fraction of void volume in rapid gravity flows of particles on a vibrated rough chute.

To determine the profiles of velocity and the fraction of void volume in a vibrated rapid shear flow of particulate solids the earlier suggested method [2] is developed.

The method consists in the analysis of an experimental particle distribution along horizontal axes during particle free falling state, which follows after the shear flow state.

This method assumes the analogy between particulate medium under conditions of relatively big component of fluctuation velocity of particles and the dense gas. Such chaotic fluctuations of particles are adequate to the thermal motion of gas molecules and have the great significance for the flow dynamics. The influence of quasithermal motion of particles on its flow dynamics is defined by determination of the «granular temperature» of particulate medium $\upsilon(y)$.

During gravity flow of particles on a vibrated rough chute this temperature is calculated with taking into account two sources of temperature generation. The first source is the quasithermal flux, generated by the vibrated chute. This flux reaches its maximum at the bed bottom and reduces towards the free surface of the bed. The second source of temperature generation is the shear flow of the bed by the influence of gravity forces. Then the total granular temperature will be defined as the following sum of temperatures

$$\upsilon(y) = \upsilon_V + \upsilon_g, \tag{1}$$

where υ_g and υ_V are the granular temperatures generated by the gravity forces and vibration respectively.

The abovementioned analogy is formulated as the interrelationship between dilatation $\bar{\varepsilon}$, normal pressure p and granular medium temperature v during rapid shear flow of nonelastic cohesionless particles as follows [2]

$$p\,\bar{\varepsilon} = \chi\,\upsilon = \chi(\upsilon_V + \upsilon_g), \tag{2}$$

where $p(y) = \int\limits_{h-y}^{h} p(y) g \cos\alpha dy$ is the hydrostatic pressure analogue, $\bar{\varepsilon} = (\varepsilon - \varepsilon_0)/(1-\varepsilon)$ is the flow dilatation, χ is the coefficient of the particulate medium state equation.

In order to determine the flow parameters in a steady state flow with due regards to eq. (2), it is necessary to establish the granular medium temperature as a function of the y – coordinate perpendicular to the shear flow direction, i. e. $\upsilon = \upsilon(y)$.

The granular medium temperature is defined as an analogue of the kinetic energy of particles caused by their mutual displacement. Three elementary types of mutual displacements of particles were taken into account [7] to define the granular medium temperature υ_g generated by the gravity shear flow in the following way

$$\upsilon_g = \upsilon_{sh} + \upsilon_{fl} + \upsilon_{tr}, \tag{3}$$

where: υ_{sh} is the energy caused by the relative velocity of particles to shear direction; υ_{fl} is the energy depending on the chaotic fluctuation of particles; υ_{tr} is the energy caused by transversal mass transfer of particles.

The calculation scheme of the temperature component υ_V, owing to the flow vibration is shown in Fig. 1, which demonstrates the countercurrent directions of the quasihydrostatic pressure p, and quasithermal flux Q, which occurs due to a bed bottom vibration.

In a steady state the quasithermal flux generated by vibrated chute towards particle bed is proportional to the bed pressure on the chute and depends on its vibration parameters.

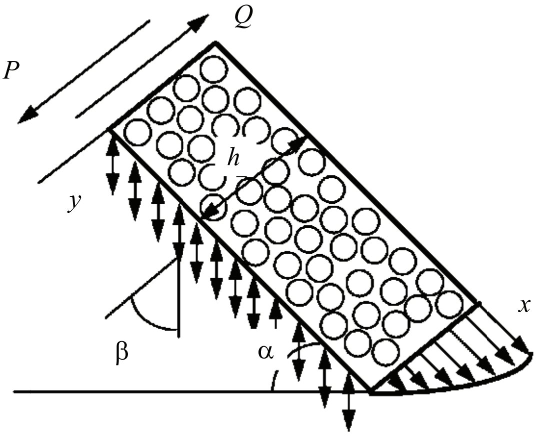

Fig. 1. Schematic of calculation of the granular medium temperature on a vibrated chute

Let us assume that the particulate solids medium consisting of non-elastic cohesionless spherical particles during rapid shear flow acquires the properties of an elastic squeezed system. During the harmonic chute vibration the periods of squeezing and dilatation follow each other. At the squeezing period the thermal energy has been brought in the bed. This energy provides the bed dilatation in the following period. At the squeezing period the particles of such medium contacting with vibrated rough chute are clung to its surface because of dispersion pressure of bed particles.

Then the reverse medium deformation dilatation will begin, when the chute will be moving in the return point. In the course of harmonic vibration this point is characterized by the maximum value of the vibration acceleration. At the sufficiently high values of the vibration acceleration $a_V > g$ in this point the particles contacting with the chute will come off from its surface and their displacement will take place as for free falling particles.

According to this approach the power chute action on the particle bed is interrupted when the time moments of its maximum squeezing extent is achieved and y – chute surface coordinate is equal to the minimum bed depth $y_{\min}$

$$t_0 = \frac{n}{\omega} + \frac{\pi}{4\pi\omega} = \frac{n + 0.25}{\omega}. \quad (4)$$

The power action of the chute on the particle bed will renew in the point of its collision. This point is determined by means of solving the two equation system, describing displacements of the chute surface and the particles coming off from surface:

$$y_{\mathrm{p}} = y_{\min} - g(t - t_0)^2 / 2\cos\alpha; \quad (5)$$

$$y_{\mathrm{ch}} = y_{\min} - A\sin(\omega(t_0 + t))\cos\beta. \quad (6)$$

The equation system (5) – (6) solving provides the time t_c and coordinate y_c determination which correspond to the initial moment of the power action of the chute.

Thus the vibrated chute has generated the quasithermal flux into the rapid gravity flow of particles during time periods $nT + t_c$, $nT + t_0$. The flux value is calculated with taking into account physical and mechanical properties of the chute surface and particles, vibration parameters also structural and kinematical flow characteristics.

The quasithermal flux will be determined as the sum of normal and tangent components of power action and energy dissipation flux

$$Q(h) = Q_n(h) + Q_t(h) - Q_d(h). \quad (7)$$

The normal flux component is calculated as follows

$$Q_n(h) = \int_{t_c}^{t_0} \omega p_y(h) \upsilon_y(t)dt, \quad (8)$$

where $\upsilon_y(t) = A\omega\cos(\omega t)\cos\beta$.

The tangent flux component is defined in the following way

$$Q_t(h) = \int_{t_c}^{t_0} \omega\, p_y(h)\mu(h)\upsilon_{x,\mathrm{rel}}(h,t)dt, \quad (9)$$

where $\mu(h)$ is the dynamic friction coefficient determined as the ratio of tangent and normal stresses.

The relative tangent velocity between particles besides the chute surface is calculated as follows

$$\upsilon_{x,\mathrm{rel}}(h,t) = \frac{dU_x(h)}{dy} b(h)d(h) - \upsilon_x(t),$$

where $\upsilon_x(t) = A\omega\cos(\omega t)\sin\beta$.

The energy dissipation component $Q_d(h)$ may be determined with taking into account physical and mechanical properties of the colliding chute surface and particles and their collision conditions on the basis of combination of the two basic hypothesises of the impact momentum transfer during collisions [8], namely: Rausse and λ – hypothesizes

$$Q_d(h) = \omega b^{-2} d^{-2}(h) m(\mu(1 + k)j\sin^2\gamma + \lambda q\cos^2\gamma + (1 - k)\omega)^2/2, \quad (10)$$

where: μ, k, λ are the coefficients of friction, restitution and tangent velocity reduction at collisions determined at the flow conditions on the chute surface; j, q are the values of the relative velocity components before collision along the impact line and tangent direction respectively; γ is the angle between the impact line and the relative velocity vector of collision.

The components j and q are calculated as follows

$$J = A\omega\cos(\omega t_c)\cos\beta + g(t_c - t_0)\cos\alpha;$$

$$q = A\omega\cos(\omega t_c)\sin\beta + g(t_c - t_0)\sin\alpha.$$

Evidently, at the relatively low vibration amplitude $A \le s$ the quasithermal flux, generated by the bed bottom in a particulate medium, will be proportional to the concentration of particles and their collision frequency with the bottom, i. e.

$$Q(y) = b^{-2}(y)d^{-2}(y)\ \upsilon_V(y)F(y), \quad (11)$$

where $F(y) = \upsilon'(y)/s(y)$ is the collision frequency of particles.

Obviously, according to the mechanical energy dissipation in the bed volume the granular medium temperature decreases towards the bed free surface. In order to calculate the granular medium temperature as an y-coordinate function, we have determined the corresponding quasithermal flux as follows [2]

$$Q(y) = q(h) - \int_{h-y}^{h} \Delta Q(y)dy, \quad (12)$$

where $\Delta Q(y)$ is the specific dissipation energy per volume unit of the bed. The specific dissipation energy is calculated in the following way

$$\Delta Q(y) = E(y)(b(y)d(y))^{-3}F(y), \quad (13)$$

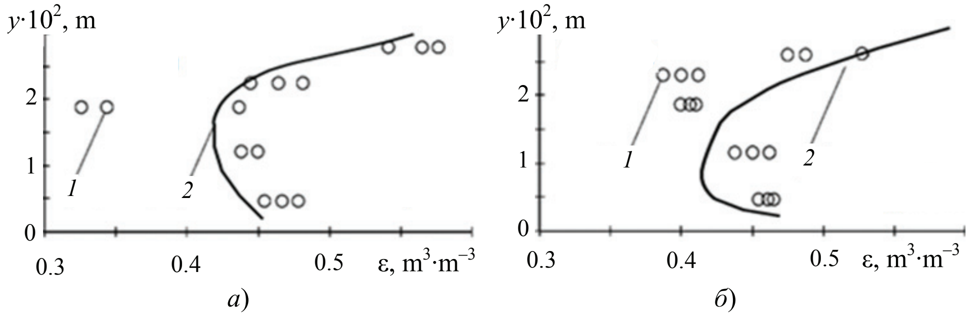

Fig. 2. Experimental (*1*) and calculated (*2*) profiles of fraction of void volume in the vibrated motionless bed

(for vibration frequency $f = 50$ Hz, and accelerations: $a_V = 2g(a)$ and $a_V = 4g(b)$)

where $E(y) = \frac{1}{2} m \left[\frac{1}{2}(1-k^2) + \frac{\pi}{32}\mu^2 (1+k)^2 + \frac{\pi}{8}\overline{\lambda}^2 - \mu(1+k) - \frac{\pi}{8}\mu\overline{\lambda}(1+k) \right] \left(\frac{v'}{2} \right)^2$ is the specific dissipation energy in terms of one particle collision.

On the other hand, the quasithermal flux may be expressed in accordance with Eq. (11) as follows

$$Q(y) = b^{-2}(y) \frac{d^{-2}(y) m (v'(y))^2}{2} \frac{v'(y)}{s(y)}. \tag{14}$$

Then, using Eqs. (12) and (14), we can calculate the fluctuation velocity $v'(y)$ and as a result to determine the granular medium temperature, which takes place due to the bed bottom vibration, in the follow way

$$v_V(y) = \frac{m (v'(y))^2}{2}. \tag{15}$$

Further, this temperature may be used to define the profiles of velocity $u(y)$ and the fraction of void volume $\overline{\varepsilon}(y)$ in a vibrated gravity flow on the basis of the experimental and analytical method [9] using Eq. (2).

In order to check directly the adequacy of the method suggested the comparison of the modeling and experimental profiles of fraction of void volume in the bed of glass beads of fraction +3.25…–3.5 mm on a vibrated horizontal bottom was carried out.

The experiment was performed using a horizontal cell of sizes 110×170 mm, installed on the vibration generator. The vibration parameters were changed during experiment.

The fraction of void volume along the bed depth was measured by means of the use of the corresponding cross sectional shearing partitions.

The comparison of the modeling results and experimental data presented in Fig. 2, shows their qualitative analogy. However, there is some quantitative inadequacy the reasons of which have to be investigated additionally before a further study of segregation in vibrated gravity flows of particulate solids with the method performed.

Finally, it is noteworthy, that there are some gradients for solid phase concentration distribution along the bed depth on Fig. 2. Thereby, in the consequence of different gradient directions in the upper and lower bed parts will take place quasidiffusional fluxes (migration effects) also of different directions [10]. This result can be used as an hypothetic explanation of Brazil nut problem (**BNP**) [3].

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Прогнозирование виброреологических эффектов во время быстрого гравитационного течения твердых частиц

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Ключевые слова и фразы: вибрация; гравитационный поток; сдвиговые потоки; шероховатый скат.

Аннотация: Рассмотрены экспериментальный и аналитический методы прогнозирования профилей скорости и порозности в быстром гравитационном потоке твердых частиц на вибрирующем шероховатом скате. Методы предполагают провести аналогию между зернистой средой в условиях относительно большой составляющей скорости пульсации частиц и плотным газом. На первом этапе методы применены для случая вибросжиженного слоя неэластичных несвязанных частиц.

Prognostizierung der vibrorhelogischen Effekte während der schnellen Gravitationsströmung der festen Teilchen

Zusammenfassung: Es ist die experimentale und analytische Methode der Prognostizierung der Profile der Geschwindigkeit und der Porosität im schnellen Gravitationsstrom der festen Teilchen auf dem vibrierenden unebenen Abhang angeboten. Die Methode vermutet die Analogie zwischen der körnigen Umgebung in den Bedingungen der großen bildenden Geschwindigkeit des Pulsierens der Teilchen und dem dichten Gas. In der ersten Etappe ist die Methode für den Fall der Vibroschicht der nicht elastischen unzusammenhängenden Teilchen erprobt.

Prévision des effets vibrorhéologiques lors de l'écoulement gravitationnel rapide des particules solides

Résumé: Est proposée le méthode expérimentale et analytique de la prévision des profils de la vitesse et de la porosité dans un écoulement gravitationnel rapide des particules solides sur une pente raboteuse vibrante. La méthode suppose une analogie entre le milieu granuleux dans les conditions de la composante de la vitesse de la pulsation relativement grande et le gaz compact. Pendant la première phase la méthode a été éprouvée pour le cas de la couche vibroliquéfiée des particules inélastiques non liées.

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