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# DYNAMIC LINEARIZATION OF STIFFNESS CHARACTERISTICS OF ELASTIC BEARINGS WITH RADIAL CLEARANCE OF LOADED ROTOR

**V.I. Galaev, O.V. Lomakina**

*Tambov State Technical University, Tambov*

*Represented by Doctor of Technical Sciences, Professor V.F. Pershin*

**Key words and phrases**: amplitude oscillations; the equivalent stiffness; the frequency; the radial gap; the reference node; the resonance and the skeletal surface; rotary machine.

**Abstract**: The paper studies the calculating dependence of the equivalents stiffness characteristics in both horizontal and vertical directions of the node references with the clearances of the rotor under the action of technological stress. The results make it possible to conduct a qualitative investigation of the tasks of nonlinear dynamics of rotor systems.

The development and construction of rotary machine design should take into account the requirements to provide an acceptable level of vibration, which is a testament to its excellence and serves as a guarantee of durability and reliability, since work flow machine and vibration generated by it are closely linked and interdependent. Rotary machine is a multi-parameter dynamic system containing nonlinear relationships under the influence of loads, differing both in nature and intensity of action.

The main working body of the machines of this type is a rotor, rotating in the elastic supports. It should be noted that the motion of the rotor in horizontal and vertical planes are interconnected, because of the gyroscopic members of differential equations of motion of the rotor, and the presence of radial gaps in its support, formed due to wear pins and bearings of the rotor.

The need to study the vertical oscillations of the rotor is predetermined by the problem of calculating dynamic forces transmitted to the body of the car by the rotor [1]. Vibrations of the rotor in the horizontal plane are determined for certain types of rotating machinery quality and precision of manufacturing operation. The examples of such machines are the ones used in leather processing, intended for semi-finished leather with required thickness and a smooth clean surface.

The problem of determining the equivalent linear stiffness characteristics of reference sites with gaps loaded rotor, which makes it possible to conduct

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Галаев Валентин Иванович – кандидат технических наук, доцент кафедры «Теоретическая механика», e-mail: tolmat@nnn.tstu.ru; Ломакина Ольга Владимировна – кандидат педагогических наук, доцент кафедры «Теоретическая механика», e-mail: lomakinaolga@mail.ru, ТамбГТУ, г. Тамбов.

some studies of dynamic processes based on the linear differential equations, taking into account the qualitative features of «rotor – elastic supports with a clearance» nonlinear system is considered.

The dynamic model of the rotor, rotating in the elastic supports with gaps, is shown in Fig. 1. where $m$, $C_y$, $C_z$ equal to mass of rotor, overall rigidity of the shell reference nodes in the horizontal and vertical directions respectively, $\delta(\alpha)$ is a radial clearance in the bearings of the rotor as a function of the angle of deviation of its pins from the vertical direction, $\alpha_{\text{st}}$ is a static deflection angle due to the influence of the technological burden on the rotor, $\alpha$ is a dynamic angle of the rotor pins, $f = \dfrac{F_z}{F_y}$ is the ratio between vertical and horizontal components of the technological burden, $y_{\text{st}}$, $z_{\text{st}}$ are the static displacements of the center of mass of the rotor in horizontal and vertical directions, respectively, due to deformation of the support units, $y_{\text{a}}$, $z_{\text{a}}$ are absolute dynamic displacement of the center of mass of the rotor in that direction.

The potential energy of the «rotor – an elastic support with a gap» system is

$$\Pi = \frac{1}{2}C_y[y_{\text{a}} + y_{\text{st}} - \delta(\alpha+\alpha_{\text{st}})\sin(\alpha+\alpha_{\text{st}}) + \delta(\alpha_{\text{st}})\sin\alpha_{\text{st}}]^2 +$$
$$+\frac{1}{2}C_z[z_{\text{a}} + z_{\text{st}} - \delta(\alpha+\alpha_{\text{st}})\cos(\alpha+\alpha_{\text{st}}) + \delta(\alpha_{\text{st}})\cos\alpha_{\text{st}}]^2 -$$
$$-mg[z_a + z_{\text{st}} + \delta(\alpha_{\text{st}})\cos\alpha_{\text{st}}].$$

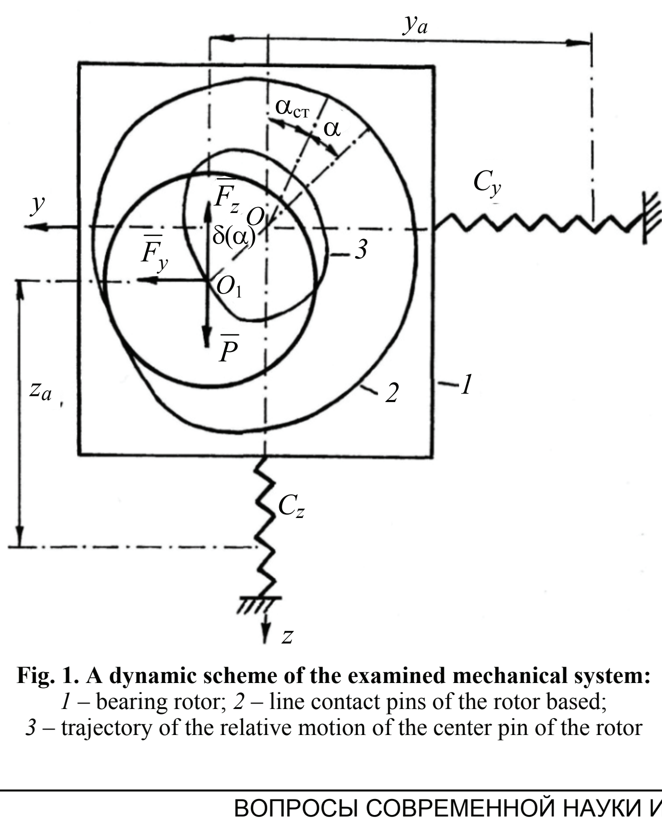


**Fig. 1. A dynamic scheme of the examined mechanical system:**
*1* – bearing rotor; *2* – line contact pins of the rotor based;
*3* – trajectory of the relative motion of the center pin of the rotor

The kinetic energy of the rotor is calculated by the formulae

$$T = \frac{1}{2} m\left(\dot{y}_{\text{a}}^2 + \dot{z}_{\text{a}}^2\right) + \frac{1}{2} A\omega^2,$$

where $A$ is a polar moment of inertia rotor; $\omega$ is its angular velocity.

Generalized forces corresponding to non potential forces and coordinates, $y_{\text{a}}$, $z_{\text{a}}$ and $\alpha$, for the considered mechanical system have the form:

$$Q_{y_{\text{a}}} = F_y;\; Q_{z_{\text{a}}} = -F_z;$$

$$Q_\alpha = F_y \{ [\delta(\alpha+\alpha_{\text{st}})\cos(\alpha+\alpha_{\text{st}}) + \delta'(\alpha+\alpha_{\text{st}})\sin(\alpha+\alpha_{\text{st}})] +$$

$$+ f[\delta(\alpha+\alpha_{\text{st}})\sin(\alpha+\alpha_{\text{st}}) - \delta'(\alpha+\alpha_{\text{st}})\cos(\alpha+\alpha_{\text{st}})]\},$$

where $\delta'(\alpha+\alpha_{\text{st}})$ is the derivative of the radial clearance on the corner $\alpha$.

The equations for the dynamics of the system have the following form:

$$\begin{cases} m\ddot{y}_{\text{a}} + C_y y_{\text{a}} - C_y \delta(\alpha+\alpha_{\text{st}})\sin(\alpha+\alpha_{\text{st}}) + C_y \delta(\alpha_{\text{st}})\sin\alpha_{\text{st}} = 0; \\ m\ddot{z}_{\text{a}} + C_z z_{\text{a}} - C_z \delta(\alpha+\alpha_{\text{st}})\cos(\alpha+\alpha_{\text{st}}) + C_z \delta(\alpha_{\text{st}})\cos\alpha_{\text{st}} = 0; \\ mg[\delta(\alpha+\alpha_{\text{st}})\sin(\alpha+\alpha_{\text{st}}) - \delta'(\alpha+\alpha_{\text{st}})\cos(\alpha+\alpha_{\text{st}})] - \\ - C_y[y_{\text{a}} - \delta(\alpha+\alpha_{\text{st}})\sin(\alpha+\alpha_{\text{st}}) + \delta(\alpha_{\text{st}})\sin\alpha_{\text{st}}] \times \\ \times [\delta'(\alpha+\alpha_{\text{st}})\sin(\alpha+\alpha_{\text{st}}) + \delta(\alpha+\alpha_{\text{st}})\cos(\alpha+\alpha_{\text{st}})] + \\ + C_z[z_{\text{a}} - \delta(\alpha+\alpha_{\text{st}})\cos(\alpha+\alpha_{\text{st}}) + \delta(\alpha_{\text{st}})\cos\alpha_{\text{st}}] \times \\ \times [\delta(\alpha+\alpha_{\text{st}})\sin(\alpha+\alpha_{\text{st}}) - \delta'(\alpha+\alpha_{\text{st}})\cos(\alpha+\alpha_{\text{st}})] = \\ = F_y \{[\delta(\alpha+\alpha_{\text{st}})\cos(\alpha+\alpha_{\text{st}}) + \delta'(\alpha+\alpha_{\text{st}})\sin(\alpha+\alpha_{\text{st}})] + \\ + f[\delta(\alpha+\alpha_{\text{st}})\sin(\alpha+\alpha_{\text{st}}) - \delta'(\alpha+\alpha_{\text{st}})\cos(\alpha+\alpha_{\text{st}})]\}. \end{cases} \tag{1}$$

The equation for determining the angle $\alpha_{\text{st}}$ obtained from the third equation (1), if it takes $\alpha = 0$. After the linearization of the equation of this system, we get:

$$m\ddot{y}_{\text{a}} + C_{\text{equ}}^{y} y_{\text{a}} + C_{\text{equ}}^{yz} z_{\text{a}} = 0;$$

$$m\ddot{z}_{\text{a}} + C_{\text{equ}}^{z} z_{\text{a}} + C_{\text{equ}}^{zy} y_{\text{a}} = 0. \tag{2}$$

According to the equations (2) the elastic supports with a clearance in the first approximation can be regarded as a support to the following linear elastic characteristics in the horizontal and vertical directions:

$$F_y = C_{\text{equ}}^{y} y_{\text{a}} + C_{\text{equ}}^{yz} z_{\text{a}};\quad F_z = C_{\text{equ}}^{z} z_{\text{a}} + C_{\text{equ}}^{zy} y_{\text{a}},$$

where

$$C_{\text{equ}}^{y} = \frac{C_y\left[mgr + \cos^3\alpha_{\text{st}}\left(\theta + fzz^2 C_z\right)\right]}{\Delta};$$

$$C_{\text{equ}}^{z}=\frac{C_z\left[mgr+\cos^3\alpha_{\text{st}}\left(\theta+fz\theta^2C_y\right)\right]}{\Delta};$$

$$C_{\text{equ}}^{yz}=C_{\text{equ}}^{zy}=\frac{C_yC_z\cos^3\alpha_{\text{st}}\theta z(\theta+fz)}{\Delta};$$

$$r=\delta^2(\alpha_{\text{st}})+2\delta'(\alpha_{\text{st}})^2-\delta(\alpha_{\text{st}})\delta''(\alpha_{\text{st}});\quad \theta=\delta(\alpha_{\text{st}})+\delta'(\alpha_{\text{st}})\text{tg}\alpha_{\text{st}};$$

$$z=\delta(\alpha_{\text{st}})\text{tg}\alpha_{\text{st}}-\delta'(\alpha_{\text{st}});\;\; \Delta=mgr+\cos^3\alpha_{\text{st}}(\theta+fz)\left(C_zz^2+C_y\theta^2\right).$$

Quantities $C_{\text{equ}}^{y}$, $C_{\text{equ}}^{z}$, $C_{\text{equ}}^{yz}$ are equivalent stiffness characteristics of the support with a gap, and the stiffness reflects the relationship between the movements of the rotor in horizontal and vertical planes. Thus, in the first approximation it is not possible to study the dynamics of the rotor as linear elastic supports with the above characteristics.

Equation (2) will not be linked, if $C_{\text{equ}}^{yz}=0,$ and then it is possible in the following cases:

1) radial clearance in the bearings is equal to zero, while $C_{\text{equ}}^{y}=C_y$, $C_{\text{equ}}^{z}=C_z$;

2) radial clearance and the constant angle $\alpha_{\text{st}}=0,$ that corresponds to the idle mode of the rotor (radial displacement of the rotor in a vertical direction relative to the support, in this case a small quantity of higher order than its movement in the horizontal direction), while $C_{\text{equ}}^{z}=C_z$;

3) radial clearance and constant angle $\alpha_{\text{st}}=90^\circ$, which corresponds to the work of the rotor at the selected technological load gaps (radial movement of the rotor in the horizontal direction, a small quantity of higher order than in the vertical), while $C_{\text{equ}}^{y}=C_y$;

4) value $z=0,$ that corresponds to the radial displacement of the rotor with respect to the support in the neighborhood of the point at which the tangent to the trajectory of the relative motion of the center pin is horizontal (a generalization of Case 2), while $C_{\text{equ}}^{z}=C_z$;

5) value $\theta=0,$ the radial displacement of the rotor occurs in the neighborhood in which a specified in Section 4 the tangent is vertical (a generalization of Case 3), while $C_{\text{equ}}^{y}=C_y$;

6) value $\theta+fz=0$, which corresponds to a radial displacement of the rotor relative to the support around the point, in which the normal to the trajectory of the relative motion of the center pin of the rotor is directed along the line of action of resultant forces $F_r$ и $F_b$ (specified displacement of the rotor with respect to the support is much less than its movement with the support). It should be noted that in this case $C_{\text{equ}}^{y}=C_y$ and $C_{\text{equ}}^{z}=C_z$, was expected.

It is known that the frequency of free oscillations, and hence the equivalent stiffness characteristics of nonlinear mechanical systems depends on the amplitude of oscillations [2]. To obtain these dependencies in the expansion of trigonometric functions in the equations of system (1) we need to take into account higher order terms with respect to the angle $\alpha$.

Let $y_a = A_y \sin \omega_1 t$, $z_a = B_z \cos \omega_1 t$, where $A_y$ and $B_z$ is the absolute amplitude of vibration of the rotor in horizontal and vertical directions $\omega_1$ is the oscillation frequency. The assumption that absolute motion trajectory of the center of mass of the rotor is close to elliptical is confirmed by the practice of exploitation of rotors and theoretical studies of their dynamics [1].

Equivalent stiffness of the support nodes of the rotor is determined in accordance with the formulas:

$$\begin{aligned} C_{\text{equ}}^{y} &= \frac{\omega_1}{\pi A_y} \int_0^{\frac{2\pi}{\omega_1}} \Phi_y(y_a, z_a) \sin \omega_1 t dt; \\ C_{\text{equ}}^{z} &= \frac{\omega_1}{\pi B_z} \int_0^{\frac{2\pi}{\omega_1}} \Phi_z(y_a, z_a) \cos \omega_1 t dt, \end{aligned} \tag{3}$$

where $\Phi_y(y_a, z_a)$, $\Phi_z(y_a, z_a)$ are thr coordinate functions $y_a$, $z_a$ are determined from the system of differential equations (1).

The results of integration in (3) allow us to obtain the specified stiffness as:

$$\begin{aligned} C_{\text{equ}}^{y} &= \frac{C_y\left(mgr + C_z \cos \alpha_{\text{st}} b z^2\right)}{\Delta} + \frac{C_y^2 \cos \alpha_{\text{st}} \psi \theta b^3 \left(C_y^2 \theta^2 A_y^2 + C_z^2 z^2 B_z^2\right)}{8\Delta^3}; \\ C_{\text{equ}}^{z} &= \frac{C_z\left(mgr + C_y \cos \alpha_{\text{st}} b \theta^2\right)}{\Delta} + \frac{C_z^2 \cos \alpha_{\text{st}} \psi z b^3 \left(C_y^2 \theta^2 A_y^2 + C_z^2 z^2 B_z^2\right)}{8\Delta^3}, \end{aligned} \tag{4}$$

where $b = \cos^2 \alpha_{\text{st}} (\theta + fz)$; $\psi = \delta(\alpha_{\text{st}}) + 3\delta'(\alpha_{\text{st}}) \text{tg} \alpha_{\text{st}} - 3\delta''(\alpha_{\text{st}}) - \delta'''(\alpha_{\text{st}}) \text{tg} \alpha_{\text{st}}$; $\xi = \delta(\alpha_{\text{st}}) \text{tg} \alpha_{\text{st}} - 3\delta'(\alpha_{\text{st}}) - 3\delta''(\alpha_{\text{st}}) \text{tg} \alpha_{\text{st}} + \delta'''(\alpha_{\text{st}})$.

The analysis of (4) allows us to formulate the conclusions similar to those previously considered special cases of the formulas of the first approximation for the equivalent stiffness of the rotor bearing assemblies.

From these relations it follows that equivalent stiffness is interrelated not only through the stiffness $C_y$, $C_z$ corps support units of the rotor, but also by the amplitude $A_y$, $B_z$ of its absolute radial oscillations.

The results could be illustrated by the problem of forced oscillations of the rotor poles with gaps, arising from its static unbalance. Considering the rotor as linear elastic supports, which are equal to the rigidity $C_{\text{equ}}^{y}$, $C_{\text{equ}}^{z}$, we write the wave equation in the form

$$\begin{cases} m\ddot{y}_a + C_{\text{equ}}^{y} y_a = m\varepsilon\omega^2 \sin \omega t; \\ m\ddot{z}_a + C_{\text{equ}}^{z} z_a = m\varepsilon\omega^2 \cos \omega t, \end{cases} \tag{5}$$

where $\varepsilon$ is the eccentricity of the rotor, $\omega$ is the angular velocity of rotor.

For the amplitudes of the oscillations of the system (5) we obtain:

$$A_y = \frac{m\varepsilon\omega^2}{C_{\text{equ}}^y - m\omega^2}; \quad B_z = \frac{m\varepsilon\omega^2}{C_{\text{equ}}^z - m\omega^2}. \tag{6}$$

In (6) values $C_{\text{equ}}^y$, $C_{\text{equ}}^z$ depend on the amplitudes $A_y$, $B_z$, so they should be regarded as a system of equations for these amplitudes.

Resonant, $P_{1y}$, $P_{2y}$, $P_{1z}$, $P_{2z}$ and $C_y$, $C_z$ the skeleton, the surface of the system, built on the assumption of independent changes in amplitudes $A_y$ and $B_z$ in accordance with (6) and relations $C_{\text{equ}}^y = m(\omega_c^y)^2$, $C_{\text{equ}}^z = m(\omega_c^z)^2$, are shown in Fig. 2. The values $\omega_c^y$, $\omega_c^z$ are the frequencies of free oscillations of the system. Skeletal surfaces are elliptic paraboloid.

Resonance and the skeletal surfaces, corresponding to vibrations of the rotor in horizontal and vertical planes, are built in different coordinate octants. This figure also shows the planes $C_y^{\text{L}}$, $C_z^{\text{L}}$, which are skeletal surfaces of the linear system, when in terms of equivalent rigidities only the first terms are taken into account, proving that the frequency of free vibrations of a linear system does not depend on the oscillation amplitudes.

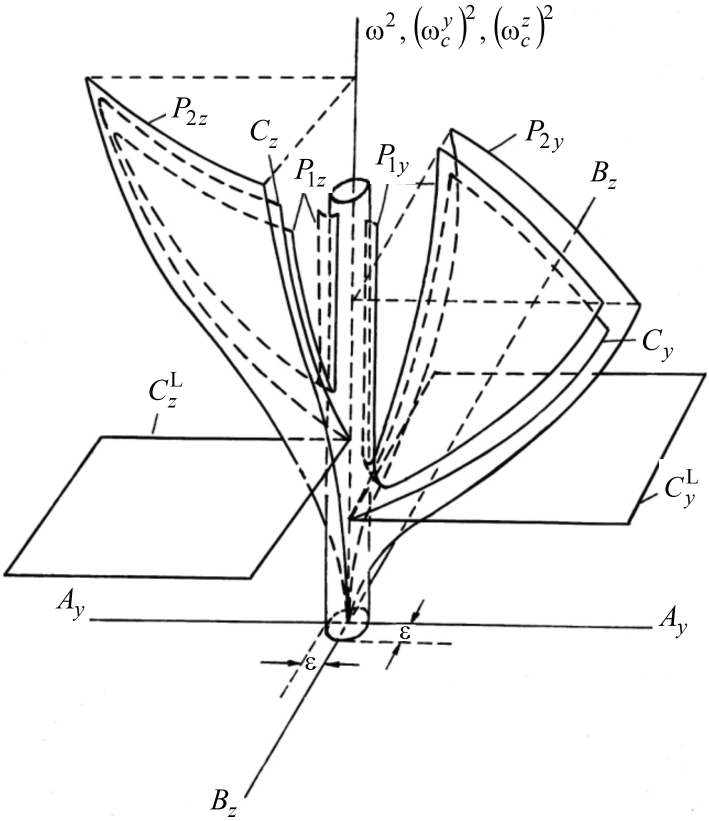


**Fig. 2. Graphical representation of the dynamic characteristics of the absolute vibration of the rotor in the bearings with clearances**

Since the amplitude of forced vibration $A_y$, $B_z$ are interrelated, in order to establish their value, corresponding to a given frequency of disturbing forces $\omega_0$, we must build the plane $\omega^2 = \omega_0^2$ and on the lines of intersection of this plane with the surface $P_{1y}$, $P_{2y}$ и $P_{1z}$, $P_{2z}$ , and find relevant points with the same coordinates, which will be the vibration amplitudes of the rotor . It is possible that the same frequency will correspond to several of these points and in this case in the system can be several modes of vibration, while some may be unstable, and with a sustainable mode of motion, the system can switch to another one as a result of random addition of external influences. Likewise is the characteristics of nonlinear systems [3].

The growth of operating speeds of rotary machines, the requirements of the stability of their vibration characteristics and dynamic functioning stipulate the importance of research objectives and reduce fluctuations in nonlinear mechanical systems of these machines.

*References*

1. Кельзон, А.С. Динамика роторов в упругих опорах / А.С. Кельзон, Ю.П. Циманский, В.И. Яковлев. – М. : Наука, 1982. – 280 с.

2. Вульфсон, И.И. Нелинейные задачи динамики машин / И.И. Вульфсон, М.З. Коловский. – Л. : Машиностроение, 1968. – 284 с.

3. Вибрации в технике. Справочник. В 6 т. Т. 2. Колебания нелинейных механических систем / под ред. И.И. Блехмана. – М. : Машиностроение, 1979. – 351 с.

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## Динамическая линеаризация жесткостных характеристик упругих опор с радиальными зазорами нагруженного ротора

**В.И. Галаев, О.В. Ломакина**

*ГОУ ВПО «Тамбовский государственный технический университет», г. Тамбов*

**Ключевые слова и фразы:** амплитуда; колебания; опорный узел; радиальный зазор; резонансная и скелетная поверхности; роторная машина; частота; эквивалентная жесткость.

**Аннотация:** Получены расчетные зависимости эквивалентных жесткостных характеристик в горизонтальном и вертикальном направлениях опорных узлов с зазорами ротора, находящегося под действием технологических нагрузок. Результаты позволяют проводить качественное исследование некоторых задач нелинейной динамики роторных систем.

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