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CUTTING VALUES DECISION MAKING FOR EXTENDED MACHINING PROCESS QUALITY

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Key words and phrases: cutting speed; optimisation of cutting values; single objective optimisation problem; single-tool turning operation.

Abstract: In practice solution results of the cutting values optimisation problem are often not flexible enough to choose for the concrete decision making in the workshop. Beyond single set of the optimal cutting values f^*, v_c^*, there are ranges of vital interest around them, i.e., reasonable decision making intervals. Based on Koch (1988), a new approach to get better, flexible solutions for the machining problem is presented. The original general and a simplified mathematical models for the decision making scope with predetermined risk are formulated. For a known example the optimum ranges for the simplified mathematical model are obtained and introduced.

Nomenclature

A – area of the rectangle in Figure 6;
c – minimization of manufacturing costs (dollars);
c_t – constant in tangential cutting force equation;
c^* – optimum value of manufacturing costs (dollars);
c_{min} – minimum value of manufacturing costs (dollars);
E – constant in modified tool life equation;
f – feed (inches/revolution);
f^* – optimum value of feed (inches/revolution);
f_{min}, f_{max} – minimum and maximum available feed (inches/revolution);
f', f'' – decision making ranges for feed;
$f^{\#}$, $f^{\#\#}$ – interval bounds for feed;
F_t max – maximum allowable cutting force (lbs);
k – predetermined risk for manufacturing costs (dollars);
l – distance traveled by the tool in making a turning pass (inches);
l – restriction for tool life limit;
n – the slope of the Taylor line on log-log paper;
n_1 – constant in modified tool life equation;
R_T – tool life limit;
r – predetermined risk in tool life limit;
T_d – tool changing time (minutes);
T_L – nonproductive time (loading, unloading and inspection time in minutes);
v_c – cutting speed (surface feet/minute);
v_c^* – optimum value of cutting speed (surface feet/minute);
$v_{c\ min}$, $v_{c\ max}$ – minimum and maximum available cutting speed (surface feet/minute);
v_c', v_c'' – decision making ranges for cutting speed (surface feet/minute);
$v_c^{\#}$, $v_c^{\#\#}$ – interval bounds for cutting speed;
x – labor plus overhead cost rate (dollars/minute);
y – tool cost per cutting edge (dollars);
α – constant in tangential cutting force equation;
β, δ – constants of the stable cutting region constraint;
γ – constant in tangential cutting force equation;
$\lambda = 12/(\pi D)$.

Introduction

The determination of optimal cutting conditions is of vital meaning to ensure maximum technical and economic efficiency in part production obtained by means of manpower and machine tools available. The rationality and economy of manufacturing, which are a result of material and energy saving and shorter machining times, depend to a large extent on the right choice of selected cutting conditions and required product quality.

Once the operation sequences and the appropriate tools have been determined, success and quality of the machining process depends on the selection of cutting values. For example, in the case of turning, cutting values include feeds and cutting speeds. The cutting values are selected to achieve the desirable performance such as good surface finish, dimensional accuracy of the component, easy chip removal and so on. In addition, they must also satisfy an economic criterion like minimum production cost or maximum production rate. Thus, machining economics involves the optimal selection of machining parameters such as cutting speed and feed subject to certain technological constraints such as tool wear, dimensional accuracy, surface finish and machine tool capabilities.

Since the early 1950s attempts have been done and results of these attempts have been published (e.g., Boston (1951), Goranski (1963)) to follow F.W. Tailor's idea from 1906 for finding "economical speeds".

This paper is mainly dealing with the cutting values single objective optimisation problem for the single-pass, single-tool turning operation where the considerations now are restricted to cutting speed, v_c, and feed, f, only.

The experiments with applying miscellaneous mathematical models and solution approaches to the single-pass, single-tool turning operation optimisation task, showed that each of the approaches gave agreeable results.

The practice has also shown that after the optimisation problem has been solved, the operator has the solution, including values, for example, for feed and cutting speed. Usually these values are represented by some real numbers such as, for instance, $f^* = 0.052$ inch/rev, $v_c^* = 143.7$ feet/min, whereas, most machine tools for cutting operations can only deal with certain discrete natural numbers. So, the operator is left with the real single set of cutting values for the particular operation, which he can not use in practice. In this situation the operator has to be skilled and experienced enough to take a responsibility to choose some suitable discrete values for the machine tool, which are no longer the optimal ones and could entail decrease in quality of the machining process and as a result – the quality of product.

To overcome this problem is possible by, for example, buying advanced machine tool with possibilities to change continuously the feeds and cutting speeds for the operations. Such machine tools are very expensive, which leads to the situation where not every manufacturing company can afford to purchase them.

Following Koch (1988), the attempt of this report is to try to subdue the problem of choosing only the discrete numbers from the optimum cutting values, f^* and v_c^*, by introducing the ranges around them, so that the solution would be more flexible by giving operator the opportunity to choose the cutting values for the machine tool from the optimum range of the cutting values.

In this paper the realisation and testing the relevance of this idea are presented. The confirmation of the results from the numerical experiments and the production engineering consequences are discussed. Finally, conclusions and an outlook for our further work are given.

1 Decision Making Scope With Predetermined Risk

The determination of cutting values includes different methods in parallel (Fig. 1). Conventionally based methods for different applied situations are required: the generation results from actual cutting process, personal experience, or calculation results.

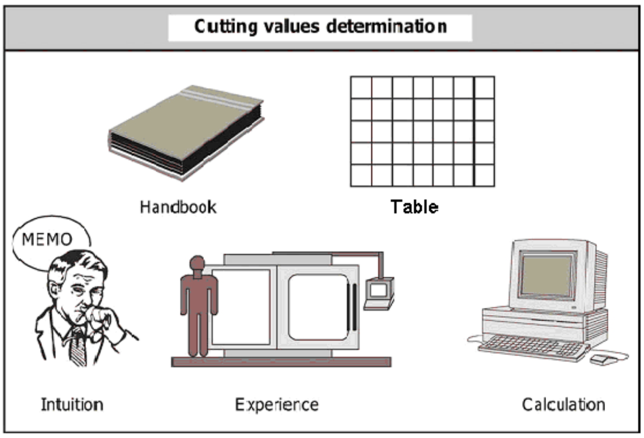

Fig. 1 Possibilities for cutting values determination

The calculation contains mathematical models for optimisation, corresponding software, experienced programmers and operators to assess the results of the calculation.

When optimisation model is formulated and approved, programmer writes the computer code for the program, runs it and gets the optimum solution points for the cutting values. However, such solution points are often not flexible enough to choose for the concrete decision making in the workshop. Around these optimal cutting values f^*, v_c^*, it is possible to determine certain variation ranges, i.e., reasonable decision making intervals, by the following approach (Koch, 1988):

If the manufacturing engineer is willing to take a predetermined risk that might, for example, consist in:

a) spending a small amount k more in manufacturing costs than the minimum costs $c^* = c(f^*, v_c^*)$ and/or

b) extending the feasible tool life limit R_T by a certain feasible level r due to existing experiences;

then a new optimisation problem can be formulated and treated.

The graphical illustration of this situation is shown in Fig. 2.

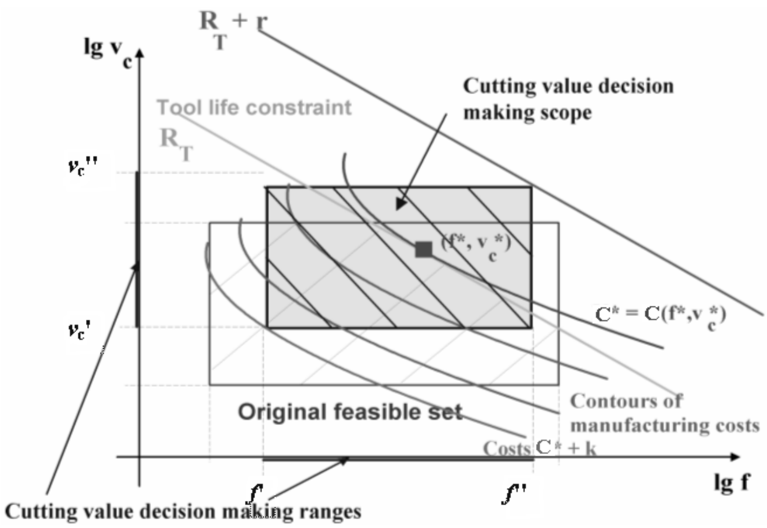

Fig. 2 Cutting values decision making intervals

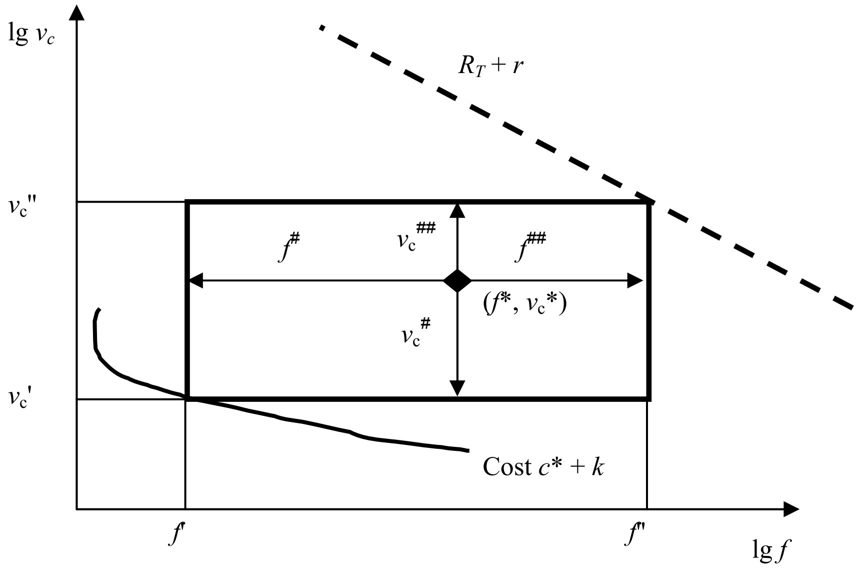

Fig. 3 Variable interval bounds

The new optimisation problem can be formulated as follows:
Determine the (variable) interval bounds (Fig. 3):

$$f^* - f' = f^{\#};$$

$$f'' - f^* = f^{\#\#};$$

$$v_c^* - v_c' = v_c^{\#\#};$$

$$v_c'' - \ v_c^* = v_c^{\#\#}$$

close to f_c*, v_c* such that the function describing the area of an inscribed rectangle gets a maximum value under the constraints that the predetermined risk (according to the given r and k) is not exceeded, i.e.:

$$\left(f^{\#} + f^{\#\#}\right)^* \left(v_c^{\#} + v_c^{\#\#}\right) \to \text{Max}!,$$

where with any point of that rectangle $[f^{\#},\ f^{\#\#}] \times [v_c^{\#},\ v_c^{\#\#}]$ it is not "allowed" to exceed

- the optimum manufacturing costs (or time) not more than $c_{\min} + k$

and

- certain restrictions l (e.g., tool life limit R_T) not more than $g_l + r_l$ (l = 1,…, m).

This leads to a minimisation problem with even the possibility of $v_c^{\#} \le 0$ or $f^{\#} \le 0$ or $v_c^{\#\#} \le 0$ or $f^{\#\#} \le 0$

$$\left(f^{\#} + f^{\#\#}\right)\left(v_c^{\#} + v_c^{\#\#}\right) \to \text{Max}! \tag{1}$$

subject to

$$c^* \le c(f^* - f^{\#},\ v_c^* - v_c^{\#}) \le c^* + k\ ; \tag{2}$$

$$c^* \le c(f^* + f^{\#\#},\ v_c^* + v_c^{\#\#}) \le c^* + k\ \ ; \tag{3}$$

$$c^* \le c(f^* - f^\#,\ v_c^* + v_c^{\#\#}) \le c^* + k\ ; \quad (4)$$

$$c^* \le c(f^* + f^{\#\#},\ v_c^* - v_c^\#) \le c^* + k\ ; \quad (5)$$

and/or

$$R_T \le R(f^* - f^\#,\ v_c^* - v_c^\#) \le R_T + r\ ; \quad (6)$$

$$R_T \le R(f^* + f^{\#\#},\ v_c^* + v_c^{\#\#}) \le R_T + r\ ; \quad (7)$$

$$R_T \le R(f^* - f^\#,\ v_c^* + v_c^{\#\#}) \le R_T + r\ ; \quad (8)$$

$$R_T \le R(f^* + f^{\#\#},\ v_c^* - v_c^\#) \le R_T + r\ . \quad (9)$$

2 Cutting Values Optimisation Task

In order to test the reasonability and relevance of the model (1) – (9) we used an existing cutting values optimisation example selected from the vast scale of published works. This example is taken from the paper published by Philipson and Ravindran (1979). A single diameter is to be turned in one pass using optimal feed and cutting speed, which will minimise costs.

The formulation of the optimisation problem, using minimum cost per component as the optimisation criterion, is presented here.

The objective function is:

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$$c = xT_L + x\frac{l}{\lambda v_c f} + \left(xT_d \frac{l}{\lambda E} + \frac{yl}{\lambda E}\right) v_c^{[(1/n)-1]} f^{[(1/n_1)-1]} \quad (10)$$

subject to

$$v_c \le v_{c\,max}\ , \quad (11)$$

$$v_c \ge v_{c\,min}\ , \quad (12)$$

$$f \le f_{max}\ , \quad (13)$$

$$f \ge f_{min}\ , \quad (14)$$

$$v_c f^\alpha \le \text{constant}\ , \quad (15)$$

$$v_c^\delta f \ge \beta\ , \quad (16)$$

$$v_c,\ \ f \ge 0\ . \quad (17)$$

The following example, given by Philipson and Ravindran (1979), is used to illustrate the solution of the optimisation problem (10) subject to the constraints (11) – (17).

A single diameter is to be turned in one pass using optimal feed rate and cutting speed which will minimize costs. The bar is 2.75 inches diameter by 12.00 inches long. The turned diameter is 2.25 inches diameter by 10.00 long. In the cutting speed calculations a mean diameter of 2.50 inches will be used. The lathe has a 15 horsepower motor and a maximum speed capability of 1500 rpm. The minimum speed available is 75 rpm. The cost rate x = \$0.15/minute, tool costs y = \$0.50, idle time T_L = 2.00 minutes, T_d = 1.00 minutes, tool life constants E = 113 420; n = 0.31; n_1 = 0.45; λ = 1.528. Maximum available cutting force F_t max = 1583.0 lbs. Other constants are as follows c_t = 344.7; α = 0.78; β = 380 000; γ = 0.9; δ = 2.0.

After inserting the fixed values into equation (10), the cost function becomes

$$c = 0.30 + \frac{982.0}{v_c f} + 8.1 \times 10^{-9} v_c^{2.333} f^{1.222}\ . \quad (18)$$

(Note: for ease of calculations, f is expressed in thousandths of an inch/revolution rather than inches/revolution).

The constraints on v_c and f are given by:

$$v_c \le 982.0\,; \tag{19}$$

$$v_c \ge 49.1\,; \tag{20}$$

$$f \le 35.0\,; \tag{21}$$

$$f \ge 1.0\,; \tag{22}$$

$$v_c f^{0.78} \le 4000.0\,; \tag{23}$$

$$v_c^{2.0} f \ge 380000.0\,; \tag{24}$$

$$v_c,\ \ f \ge 0\,. \tag{25}$$

To compute this cutting values optimisation problem the nonlinear optimisation system ***NOSYS*** (Koch, 1993, 1996) has been used. The ***NOSYS*** assists the user to solve complex optimisation tasks numerically (***NOSYS/MENOS***). In case of 2 problem variables, it also has a possibility to provide for the user the graphical optimisation by manipulation of bounds and moving through the feasible set (***NOSYS/GRANOS***).

By choosing ***GRANOS*** graphical optimisation module of the ***NOSYS*** it is practicable to perform the graphical demonstration of the results of the calculation. The graphical results of the cutting values optimisation problem are presented in the fig. 4.

The comparison of the results for the cutting values optimisation problem from the Philipson and Ravindran (1979) paper with our calculations shown in Table 1.

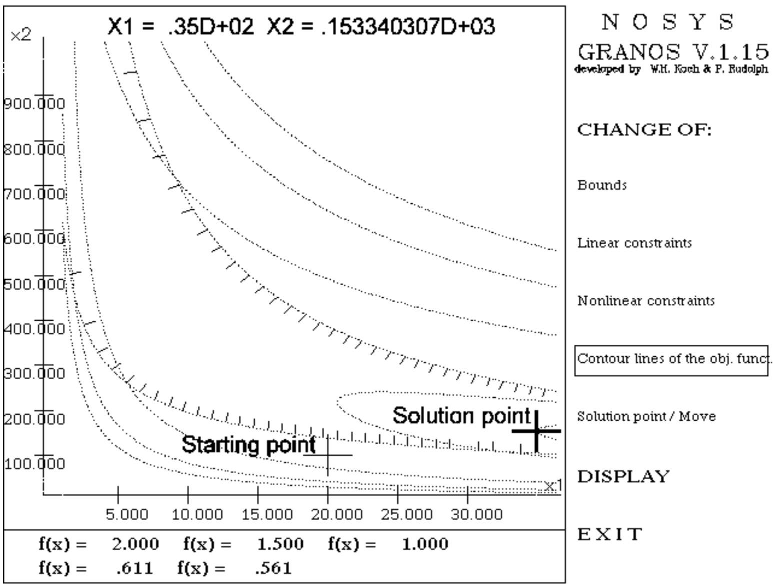

Fig. 4 ***NOSYS/GRANOS*** **representation of the solution point for the Philipson and Ravindran (1979) case,** ***X*1 = *f* and *X*2 = v_c**

Table 1

Comparison of the results

Optimal values	Philipson and Ravindran (1979)	Koch and Ponomareva (2004)
f^*, inches/revolution	0.035	0.035
v_c^*, surface feet/minute	153.37	153.33
c^*, $	0.561	0.561

This comparison of the results of Philipson and Ravindran (1979) model shows that our program works and the software tool ***NOSYS*** is reliable. Slight difference in result of the optimum value for the speed v_c^* could be due to:

- negligible differences in the precision of the software used by Philipson and Ravindran and software tool ***NOSYS*** we used;
- more powerful and precise computers and /or real numbers representation available nowadays.

3 Application of the Idea of the Decision Making Scope with Predetermined Risk to the Philipson and Ravindran Case

A deeper analysis of the situation shown in section 2 (see Fig. 4) can be carried out with ***NOSYS***/***GRANOS*** by altering the bounds to zoom into the feasible set. Fig. 5 shows the solution point $c^*(f^*, v_c^*)$ for the optimisation problem (18) subject to constraints (19) – (25) and the contour line of the objective function with the predetermined risk in manufacturing costs $c^* + k$. The predetermined risk is assumed to be 5 cents and objective function with that respect is equal to c^*+k = $0.611.

Around the neighbourhood of the solution point $(f^*, v_c^*)^T = (0.035, 153.332793)^T$ with $c^* = 0.56140127$ there are ranges of the cutting values as shown by formulation of the optimisation mathematical model (1) subject to constraints (2) – (9).

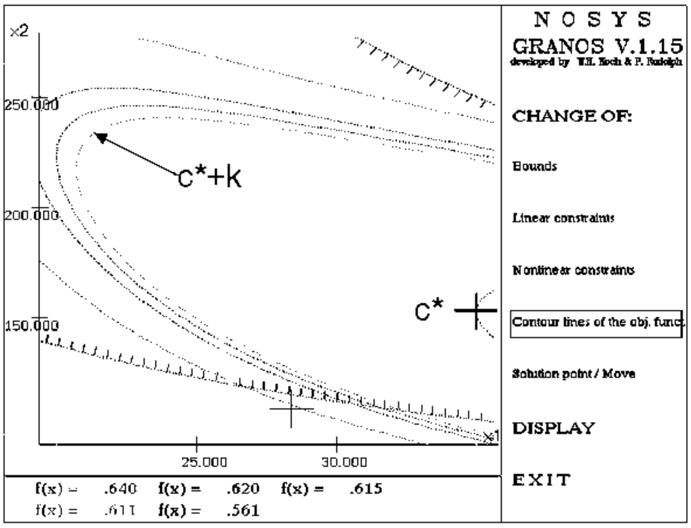

Fig. 5 Feasible set for the optimisation task

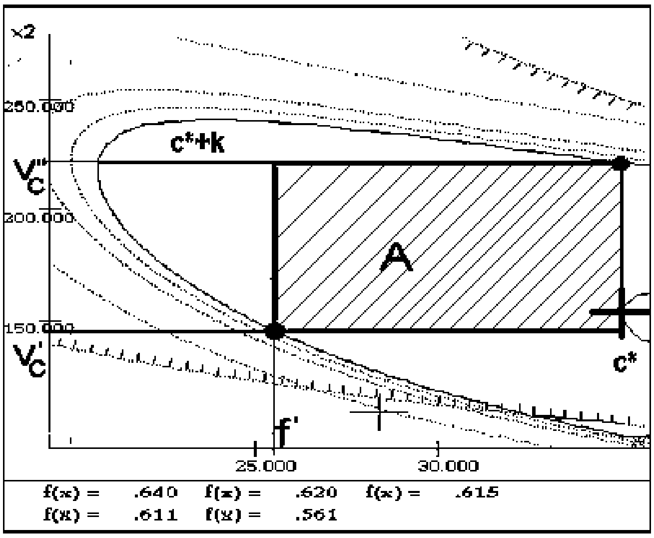

Fig. 6 Optimum ranges of the cutting values as the solution of the optimisation task (26) subject to constraints (27) and (28)

This is a general case for the decision making scope determination with the predetermined risk optimisation task.

The first attempt to approach the solution for this task has led to a simplified mathematical model. It has been decided to use only manufacturing costs objective function and instead of using several inequality constraints to use 2 equality constraints (see Fig. 6):

a) one for the point (f', v'_c) as left lower corner and

b) another one for the point (f^*, v''_c) as right upper corner of the inscribed rectangle.

This leads to the optimisation task:

$$A = ab = (f^* - f')(v''_c - v'_c) \rightarrow \text{Max!} \qquad (26)$$

subject to

$$c(f', v'_c) = c^* + k; \qquad (27)$$

$$c(f^*, v''_c) = c^* + k. \qquad (28)$$

4 Results

To solve optimisation task (26) subject to constraints (27) – (28) the corresponding computer code for ***NOSYS*** has been written. The results of the calculation are demonstrated in Fig. 6. It shows the maximum possible value of the function describing the area of the inscribed rectangle under the constraints with the predetermined risk $c^* + k$. As depicted on this figure the optimum solution $c^*(f^*, v_c^*)$ of the Philipson and Ravindran case lies on the right edge of the rectangle with the area A.

Table 2 shows the comparison of the results for the classical optimisation approach for the cutting value optimisation task given by Philipson and Ravindran (1979) and decision making scope with predetermined risk approach of Koch and Ponomareva (2004).

Table 2

Comparison of the results

Variables	Philipson and Ravindran (1979)	Koch and Ponomareva (2004)
f^*, inches/revolution	0.035	0.0254 – 0.035
v_c^*, surface feet/minute	153.37	146.6260 – 221.3298
c^*, $	0.561	0.611

The results show that the operator can now choose the cutting values for the machining operation from the range of optimum values. The optimum solution point $(f^*, v_c^*)^T = (0.035, 153.332793)^T$ with $c^* = 0.56140127$ is a part of the optimum range with its $f^* = f'' = 0.035$ lying on the edge of the rectangle with the area A; and the $v_c^* =$ $= 153.332793$ lying inside the interval v_c' and v_c''. The question whether it is a necessary condition to have the solution point to be a member of the optimum cutting values range could be a subject for further investigations.

It is always better to have flexibility in the solution for the machining operations. Having optimum ranges for the feed f and cutting speed v_c, gives an operator some room to adapt the solution to a real-world situation in the workshop. Of course, for flexibility it has to be spent the value k of 5 cents.

5 Conclusions and Outlook for Further Work

In practice a single set of cutting values of optimisation problem is often not flexible enough to choose for the concrete decision making in the workshop. Beyond the optimal cutting values f^*, v_c^*, there are ranges of vital interest around them, i.e., reasonable decision making intervals.

The paper presents an approval and its relevance to get better, flexible solutions for the machining problem. The original general and simplified mathematical models for the decision making scope with predetermined risk have been formulated.

The optimum ranges for the simplified mathematical model have been obtained and shown in Table 2. These optimum ranges give the operator more flexibility in choosing the cutting values for the real workshop situation.

Further work is to find the solution for the generally formulated mathematical problem of decision making with predetermined risk including at least cutting depth, d_c. Another question is whether it is a necessary condition to have the solution point to be a part of the optimum cutting values range.

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Повышение качества обработки металла выбором коэффициентов резания

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Ключевые слова и фразы: коэффициент скорости резания; коэффициент подачи материала; оптимизация по коэффициентам резания; однокритериальная задача оптимизации; токарная обработка при однократном прохождении резца.

Аннотация: На практике решение задачи оптимизации коэффициентов резания часто не позволяет оператору-токарю использовать его в производстве. Вокруг оптимальных значений коэффициентов резания f^*, v_c^* имеется так называемая допустимая область принятия решения. Значения, находящихся в данной области, могут быть определяющими для их успешного применения. Излагается новый подход к проблеме, базирующийся па предложении Коха (1988 г.). При этом решение получается более гибким, а у оператора-токаря появится возможность выбора оптимальных значений коэффициентов резания. Сформулированы общая и упрощенная математические модели оптимизационной задачи принятия решения в условиях риска. Представлено решение упрощенной математической модели в виде области оптимальных значений коэффициентов резания.

Steigerung der Qualität der Metallbearbeitung durch die Auswahl von Spanenskoeffizienten

Zusammenfassung: Die Lösung der Aufgabe der Optimierung von Spanenskoeffizienten darf vom Dreher-Operator in der Produktion oft nicht benutzt werden. Um die Optimalwerte von Spanenskoeffizienten f^*, v_c^* gibt es sogenannte zulässige Entscheigungsgrenze. Die in diesem Bereich vorhandenen Werte können für ihre Erfolgbenutzung konstitutiv sein. Es wird die neue auf Koch-Vorschlag (1988) gegründete Problemeinstellung dargelegt. Dabei wird die Lösung flexibler und der Dreher-Operator hat die Möglichkeit der Auswahl der Optimalwerte von Spanenskoeffizienten. Es sind die gemeinsamen und vereinfachten mathematischen Modelle der optimierten Entscheidungsaufgabe bei den Risikobedingungen formuliert. Es ist die Lösung der vereinfachten mathematischen Modelle als Bereich der Optimalwerte der Spanenskoeffizienten dargelegt.

Augmentation de la qualité du traitement du métal par le choix du coefficient de la coupe

Résumé: En pratique la solution du problème de l'optimisation des coefficients de la coupe ne permet pas souvent à l'opérateur-tourneur de l'utiliser dans la production. Autour des valeurs optimales de la coupe f^*, v_c^* il y a un soi disant demaine admissible pour l'adoption des solutions. Les grandeurs se trouvant dans ce domaine peuvent être déterminantes pour leur application réussite. On énonce une nouvelle approche envers le problème qui est basée sur la proposition de Koch (1988). Avec cela la solution est plus souple; l'opérateur-tourneur a la possibilité du choix des solutions optimales des coefficients de la coupe. Sont formulés les modèles mathématiques général et simplifié du problème de l'optimisation de la prise des décisions dans les conditions du risque. Est proposée la solution du modèle mathématique en forme du domaine des solutions optimales des coefficients de la coupe.